

Generating functions and their applications

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The content of the seminar is talking about generating functions. The generating functions are a very useful tool in combinatorics. The knowledge of an explicit formula for a generating function to compute explicitly the terms of the sequences or at least it gives a good algorithm to compute them.

The seminar will start by talking about some introductory and motivating examples, like Fibonacci numbers, Bernoulli numbers, number of partitions of a set and other examples. After them we will talk about the two fundamental kinds of generating functions associated to a sequence $(a_n)_{n \in \mathbb{N}}$

$$\begin{aligned} \text{ordinary generating function: } & \sum_{n=0}^{\infty} a_n x^n \\ \text{exponential generating function: } & \sum_{n=0}^{\infty} \frac{a_n}{n!} x^n \end{aligned} \tag{1}$$

and the relations between algebraic operations in the ring $\mathbb{C}[[x]]$ and associated sequences in both cases.

Next we will do some applications, particularly to graph theory: we will show how these techniques allow to compute the number of labelled connected graphs, 2-regular graphs, bipartite graphs. All these examples are based on the concepts of cards, decks, Hands and exponential formula (see [Wi, Ch. 3]).

At the end we will briefly shows other applications, such like finding lattice points on polyhedra, computing the the number of solutions of certain families of diophantine equations.

The main reference is [Wi]. Anyway i have found various interesting examples in [St], [Ba] for the part on polyhedra, [AD] for the part on diophantine equations.

Bibliography

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- [St] R. Stanley (2011), *Enumerative Combinatorics*.
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