Graphs defined on groups

Presentation by Melissa Zucconi
Supervised by Prof. Andrea Lucchini

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Classroom: 1BC45

This seminar is about some graphs that can be constructed starting from a given finite group G. Vertices of these graphs are elements of the group G and edges are constructed in such a way to underline some properties of the group structure of G. In particular I'll introduce and describe the power graph, the enhanced power graph, the deep commuting graph, the commuting graph and the non-generating graph associated to a given finite group G. The main purpose is not to investigate properties of these graphs individually but I'll focus on relations between them. Indeed, it turns out that the graphs mentioned above, as long as G is non abelian, form a hierarchy; in the sense that the edge set of one is contained in the one of the next. A natural question is: when can two of them be equal? In the second part of the seminar I'll answer to this question giving sufficient and necessary conditions. I'll say something also about differences: for any pair of graphs in the hierarchy, if G is a group such that two graphs are unequal, we could ask about the graph whose edge set is the difference. We could denote these by using, for example $(Com - Pow)(G)$ for the graph whose edges are those belonging to the commuting graph but not in the power graph, with similar notations in other cases. Anyway, beside few exceptions not much is known about these differences.

REFERENCES

Peter J. Cameron, Graphs defined on groups, International Journal of Group Theory (2021).