PROPERTIES OF VISCOSITY SOLUTIONS FOR STOCHASTIC CONTROL PROBLEMS

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Abstract:

This seminar will provide several results on viscosity solutions, with the purpose of applying them to stochastic control problems when lacking the assumptions for C^2 regularity. Throughout the seminar we will first briefly introduce the base theory behind stochastic control problems, the Dynamic Programming Principle and Hamilton-Jacobi Equation. Then, we will look at the standard definition and properties of viscosity solutions in a general context: they represent a "weak" notion of solution, which requires less regularity assumptions on our PDE, but still allows us to formulate classical results such as a comparison principle. A less known formulation of viscosity (super-/sub-)solutions will also be formulated through semijets.

In the second part of the seminar we will look at how viscosity solutions can be used in stochastic control theory to produce similar results in a probabilistic setting. We will see the necessary assumptions on the system's Hamiltonian, under which the value function

$$v = \sup_{\alpha,\tau} E\left[\int_0^\tau f(X_t, \alpha_t)dt + g(X_\tau)\right],$$

under the further hypothesis of being locally bounded, is a viscosity solution of the Hamilton-Jacobi Bellman variational inequality, and how we can formulate a uniqueness result for such a viscosity solution through the comparison principle.

References:

Nizar Touzi, *Optimal Stochastic Control, Stochastic Target Problems, and Backward SDE*, Springer (2013).

Huyên Pham, *Continuous-time Stochastic Control and Optimization with Financial Applications*, Springer (2009).