

Continuum percolation

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Abstract

Let $\mathbb{L}^d$ be the d -dimensional lattice, where $\mathbb{Z}^d$ is the set of all its vertices, and suppose that any edge connecting two adjacent vertices is open with probability p and closed otherwise.

In percolation theory one is mainly interested in two problems:

1. Is there a positive probability that the connected component of the origin is unbounded?
2. If such a components exists, is it unique?

After giving the definition of Poisson Point process and of Boolean model, we will discuss the main topic: continuum percolation in the Poisson boolean model.

In particular, we will study the critical phenomena in both one and higher dimensions and examine the model's behaviour which presents two phases: a subcritical phase and a supercritical one. We will also see a comparison with the phase transition in the discrete percolation model.

Finally, we the will discuss the uniqueness of the occupied component when it exists.

References

- [1] R. Meester; R. Roy (1996), *Continuum percolation*, Press Syndicate of the University of Cambridge, Cambridge.
- [2] Alessandra Bianchi (2024), *Lecture notes of Advanced stochastic processes*.