Sufficient Conditions for Viability

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Abstract

We consider dynamical systems described by differential inclusions of the form

$$\begin{cases} \dot{x}(t) \in F(t, x(t)) & \text{for a.e. } t \in [0, T]; \\ x(0) = x_0. \end{cases}$$

Where $T > 0, x_0 \in \mathbb{R}^n$ and $F\colon [0, T] \times \mathbb{R}^n \rightsquigarrow \mathbb{R}^n$. Given $K \subseteq \mathbb{R}^n$ *Viability theory* concerns with finding relations between the set-valued map F describing the dynamics and the set K that guarantee the existence of a *viable trajectory* of the differential inclusion, namely a trajectory x s.t.

$$x(t) \in K \quad \forall t \in [0, T].$$

In this presentation after introducing the *viability problem* and explaining some applications, we will first give some preliminary definitions and results on continuity concepts for set-valued maps and define the so called *contingent cone* at a point $x \in K$, $T_K(x)$, which will then allow us to state and prove the theorem giving sufficient conditions for the existence of a viable trajectiory in the simplier case of an autonomous system, i.e. when the map F does not explicitly depend on time.

References

[1] J.P. Aubin; A. Cellina (1984), *Differential Inclusions, Set-valued Maps and Viability Theory*, Springer-Verlag.

[2] J.P. Aubin (1991), *Viability Theory*, Birkhäuser.