

p-adic Galois Representations and Tate Module

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Abstract

In this presentation we consider *p-adic Galois representations*, i.e. continuous representations of the absolute Galois group $\mathbf{G}_{\mathbf{K}} := \text{Gal}(\mathbf{K}^{\text{sep}}/\mathbf{K})$ (where $\mathbf{K}$ is a field) of the form $\rho : \mathbf{G}_{\mathbf{K}} \longrightarrow GL_n(\mathbb{Q}_p)$, where $\mathbb{Q}_p$ is the field of p-adic numbers. Such objects are very useful in different branches of algebra, algebraic geometry and representation theory: for example studying 1-dimensional representations, in the so called class field theory, allow to investigate on abelian extensions of $\mathbb{Q}$; very interesting are, in particular, representations coming from algebraic varieties. During the talk we will introduce the definition of *p-adic Galois representations* showing their important connection with algebraic objects such as the ring of *p-adic integers*, and giving some examples. We will introduce the concept of *Tate module* of an abelian group and we will go through two important examples: the so called *p-adic cyclotomic character* and the *Tate module of an Elliptic curve*. After recalling some basic notions about those curves we will conclude showing the very important role played by *Tate modules* describing properties of geometric objects such as *abelian varieties*.

Bibliography

[1] Fontaine Jean-Marc, Ouyang Yi, *Theory of p-adic Galois representations*, livre en préparation, pp. 1-17, Springer.

[2] Neukirch Jurgen, *Algebraic Number Theory*, pp. 99-109, Springer, 1999.