

The Obstacle Problem.

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Abstract.

In this seminar we will study a free boundary problem, i.e. a type of elliptic nonlinear PDE, which is called the Obstacle Problem.

This problem consists in solving partial differential equations in a domain, a part of whose boundary is unknown *a priori* and must be determined as part of the problem, hence the name free boundary.

Since in general, with few exceptions, it is impossible to explicitly determine the solution and the underlying domain, the ultimate goal is to establish analytical and geometric properties for both. In fact, during this presentation, we will start by studying the properties of the solution u , arriving at finding some "strong" regularity properties.

We will then continue trying to study the free boundary, using the blow-up technique.

It will then be possible, thanks to Dirichlet's Dichotomy Theorem, to characterize the free boundary into two sets: the set of regular points and the set of singular points.

Continuing with the analysis, it will easily be demonstrated that the set of regular points is actually a smooth and regular surface; we will be able to obtain a certain regularity also for the set of singular points, using the blow-up uniqueness Theorem at singular points, exploiting Weiss and Monneau monotonicity formulas.

Our study will then conclude with some notes on the geometric structure of the set of singular points, bringing some recent results.

1 Bibliography.

1. X. Fernandez-Real and X. Ros-Oton, *Regularity Theory for Elliptic PDE*, European Mathematical Society, 2022.
2. A. Figalli, *Free boundary regularity in obstacle problems*, ETH Zürich, 2018.