The Erdős Discrepancy Problem

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Abstract

In 1932, Paul Erdős conjectured that for any sign sequence $f : \mathbb{N} \to \{-1, 1\}$, the associated *discrepancy* is infinite:

$$\sup_{n,d\in\mathbb{N}} \left| \sum_{j=1}^{n} f(jd) \right| = +\infty.$$

A proof of this conjecture was published by Terence Tao in February 2016 in the journal *Discrete Analysis* [Tao17], building upon prior contributions by the Polymath5 project—a collaborative research effort initiated in 2010 under the leadership of Timothy Gowers.

The proof to be presented here is structured around three principal components. First, a Fourier-analytic reduction, which reformulates the original problem in terms of (stochastic) completely multiplicative functions with values in the unit circle S^1. Second, a result recently established by Tao regarding the pair correlations of multiplicative functions [Tao16], specifically the logarithmically averaged, non-asymptotic variant of the Elliott conjecture. Third, an analysis of the partial sums of completely multiplicative functions that "pretend" to be modulated Dirichlet characters of the form $n \mapsto \chi(n)n^{it}$ for some small real parameter t.

This final component constitutes a generalization of an argument previously developed by the Polymath team.

References

[Sou16] Kannan Soundararajan. "Tao's Resolution of the Erdős Discrepancy Problem". In: *Bulletin of the American Mathematical Society* 53.2 (2016), pp. 299–310. DOI: 10.1090/bull/1533.

[Tao16] Terence Tao. *The logarithmically averaged Chowla and Elliott conjectures for two-point correlations*. 2016. arXiv: 1509.05422 [math.NT]. URL: https://arxiv.org/abs/1509.05422.

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