

Cluster Algebra

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Abstract

Cluster algebras were invented by Sergey Fomin and Andrei Zelevinsky [FZ02a] in the year 2000. They were introduced in a project whose aim was to develop a combinatorial approach to the results obtained by George Lusztig concerning total positivity in algebraic geometry on one hand and canonical bases in quantum groups on the other hand. During the years, numerous links have been discovered between cluster algebras and a wide range of subjects including: Poisson geometry, integrable systems, higher Teichmüller spaces, combinatorics, commutative and non-commutative algebraic geometry, and representation theory of quiver and finite dimensional algebras.

Cluster algebras are a class of commutative $\mathbb{Q}$ -algebras with a distinguished set of generators, called the *cluster variables*, grouped into overlapping subset, called the *clusters*, of constant cardinality, that is the *rank* of the cluster algebra. They are obtained via *mutations* from an *initial cluster*. The set of cluster variables may be infinite or finite. The latter case takes the name of *cluster-finite* cluster algebras.

The aim of this seminar is to introduce the notion of cluster algebra and specialize to the case arising from a starting quiver Q without loops nor 2-cycles. We will start introducing root systems and Dynkin diagrams as they will be of important use in the classification of cluster-finite cluster algebra. We will then introduce how to associate a cluster algebra to a quiver Q and some important results by Fomin and Zelevinsky, i.e. the *Laurent phenomenon*. Finally, we generalize the previous construction in order to obtain the class of *anti-symmetric cluster algebras of geometric type*, looking at the particular example of the homogeneous coordinate ring $\mathbb{C}[Gr_{2,n+3}]$ of the Grassmannian of the 2-dimensional subspaces in $\mathbb{C}^{n+3}$.

References

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