

Exotic Spheres

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Abstract

In 1904, the French mathematician Henri Poincaré asked whether every closed, simply connected 3-manifold is homeomorphic to the 3-sphere. The conjecture was finally proved at the beginning of the twenty-first century, following the work of Grigori Perelman [1, 2].

During the twentieth century, several generalisations and variants of Poincaré's question attracted the attention of many prominent mathematicians. In this seminar, we will focus on the smooth version of the problem: is every smooth manifold that is homeomorphic to a sphere necessarily diffeomorphic to the standard sphere?

In 1956, John Milnor gave a negative answer in dimension 7. In his paper *On Manifolds Homeomorphic to the 7-Sphere* [3], he constructed manifolds that are homeomorphic, but not diffeomorphic, to the standard sphere S^7 . Such manifolds are now called *exotic spheres*.

The aim of this seminar is to retrace Milnor's proof, following the modern reformulation presented in [4]. In particular, we will construct the family of manifolds $E_{i,j}$ that provides the candidates for exotic 7-spheres. We will first determine which of these manifolds are homeomorphic to S^7 , and then prove that some of them are not diffeomorphic to the standard sphere by introducing and computing Milnor's invariant.

References

- [1] G. Perelman, *The entropy formula for the Ricci flow and its geometric applications*, arXiv:math/0211159, 2002.
- [2] G. Perelman, *Ricci flow with surgery on three-manifolds*, arXiv:math/0303109, 2003.
- [3] J. Milnor, *On Manifolds Homeomorphic to the 7-Sphere*, *Annals of Mathematics* **64** (1956), no. 2, 399–405.
- [4] N. M. Eagles, J. Lane and Y. H. Lee, *Milnor's Exotic Spheres*, unpublished manuscript, 2023. Available at: https://math.berkeley.edu/~nm.eagles/expository%20papers/Exotics_Spheres.pdf.
- [5] J. W. Milnor and J. D. Stasheff, *Characteristic Classes*, *Annals of Mathematics Studies*, vol. 76, Princeton University Press, Princeton, 1974.
- [6] A. Hatcher, *Vector Bundles and K-Theory*, Version 2.2, November 2017. Available at: <https://pi.math.cornell.edu/~hatcher/VBKT/VBpage.html>.