

Periodic Homogenization of Fully Nonlinear PDEs

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The aim of this seminar is to study periodic homogenization of a certain type of fully nonlinear PDEs, having the form $u_\epsilon + H(Du_\epsilon, x, x/\epsilon) = 0$, where the term x/ϵ can be thought of as a highly oscillatory variable that satisfies the assumption of periodicity. Specifically, the objective is to prove that as $\epsilon \rightarrow 0$, the sequence of viscosity solutions u_ϵ (see [1] for the precise definition) converges in a suitable sense to a limit function u that solves what is called the “homogenized PDE” $u + \bar{H}(Du, x) = 0$, which does not depend on the oscillatory variable. Clearly, the goal is to determine what the effective operator $\bar{H}$ is and which properties it inherits from H . Finally, to prove $u_\epsilon \rightarrow u$, we use the perturbed test function method, introduced by L.C. Evans in [3]. (See also [2] for some generalizations). The seminar will also present an application of periodic homogenization to differential games and a few examples regarding some explicit PDEs (see [4]).

References

- [1] M.Bardi, I. Capuzzo-Dolcetta *Optimal Control and Viscosity Solutions of Hamilton-Jacobi-Bellman Equations* Birkhäuser., 1997
- [2] M. Bardi, *Metodi di viscosità per l'omogenizzazione di equazioni nonlineari a derivate parziali*, Lecture Notes, 2023.
- [3] L.C. Evans, *Periodic homogenisation of certain fully nonlinear partial differential equations*, Proceedings of the Royal Society of Edinburgh, 1992.
- [4] A. Bensoussan, J.L Lions, G. Papanicolaou, *Asymptotic analysis for periodic structures*, North-Holland Publishing, 1978.