

Introduction to Non Standard Analysis

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Abstract

This seminar introduces Non Standard Analysis (NSA), a mathematical model introduced by Abraham Robinson in the 1960s that settles a past debate and formalizes the idea of infinitesimals and infinite numbers into calculus. The procedure consists in defining an extension of the real numbers called hyperreal numbers (${}^*\mathbb{R}$) as the set of real sequences modulo an equivalence relation using ultrafilters. This enables an embedding of $\mathbb{R}$ into ${}^*\mathbb{R}$ with the addition of infinitesimal numbers.

The introduction of the Transfer Principle, the fundamental theorem of this framework, guarantees that first order logical statements hold true in $\mathbb{R}$ if and only if they hold true in ${}^*\mathbb{R}$. This logical result leads to the introduction of some intuitive and powerful tools in NSA and facilitates some classical proofs of standard analysis. Nevertheless, first order logic is insufficient; therefore, a similar construction can be set up to extend the idea.

References

- [1] Davis, I. (2009) An introduction to nonstandard analysis. Available at: <https://www.math.uchicago.edu/~may/VIGRE/VIGRE2009/REUPapers/Davis.pdf> (Accessed: May 2026).
- [2] Robinson, Abraham. Non-Standard Analysis. North-Holland, 1966.
- [3] Stroyan, Keith D., and Wilhelmus A. J. Luxemburg. Introduction to the Theory of Infinitesimals. Academic Press, 1976.