Le30

Friday, 29 May 2026 08:34

Es1

$x(n) = 1 \cdot y(n-2) + 1 \cdot y(n-1) - 6y(n)$

$x(n) = A$

$y(-1) = K_1 \quad y(-2) = K_2$

1) $h(n) = ?$ BIBO STABILE?

2) $y(n), n \geq 0$?

$$H(z) = \frac{1}{1 \cdot z^{-2} + 1 z^{-1} - 6} = \frac{1}{(z^{-1}-2)(z^{-1}+3)}$$

$$z^{-1}_{1,2} = \frac{-1 \pm \sqrt{1+24}}{2} = \begin{cases} 2 \\ -3 \end{cases}$$

$$H(z) = \frac{R_0}{z^{-1}-2} + \frac{R_1}{z^{-1}+3}$$

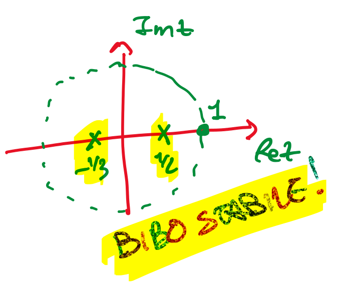
$$R_0 = H(z)(z^{-1}-2)\Big|_{z^{-1}=2} = \frac{1}{z^{-1}+3}\Big|_{z^{-1}=2} = \frac{1}{5}$$

$$R_1 = H(z)(z^{-1}+3)\Big|_{z^{-1}=-3} = \frac{1}{z^{-1}-2}\Big|_{z^{-1}=-3} = -\frac{1}{5}$$

$$H(z) = \frac{1}{5}\frac{1}{z^{-1}-2} - \frac{1}{5}\frac{1}{z^{-1}+3}$$

$P_1 = 1/2$

$P_2 = -\frac{1}{3}$



$$h(n) = -\frac{1}{5} \cdot +\left(\frac{1}{2}\right)^{n+1} 1_0(n) + \frac{1}{5} \cdot +\left(-\frac{1}{3}\right)^{n+1} 1_0(n)$$

$$x(n) = y(n-2) + y(n-1) - 6y(n)$$

$$X(z) = Y(z)z^{-2} + z^{-1}\cancel{y(-1)}K_1 + \cancel{y(-2)}K_2 + Y(z)z^{-1} + \cancel{y(-1)}K_1 - 6Y(z)$$

$$Y(z) \cdot \underbrace{(z^{-2}+z^{-1}-6)}_{(z^{-1}-2)(z^{-1}+3)} = X(z) - (K_1 + K_2 + K_1 z^{-1})$$

$$Y(z) = \frac{X(z)}{(z^{-1}-2)(z^{-1}+3)} - \frac{K_1+K_2+K_1z^{-1}}{(z^{-1}-2)(z^{-1}+3)}$$

RISP. FORZATA — EVOL. LIBERA

$x(n) = A$

$x_+(n) = A \cdot 1_0(n)$

$$+\cancel{P_0^{n+1}} 1_0(n) \xrightarrow{z} \frac{-1}{z^{-1} - \cancel{P_0}^{\,1}} = \frac{-1}{z^{-1}-1}$$

$P_0 = 1$

$$X(z) = \frac{-A}{z^{-1}-1}$$

$$Y_f(z) = \frac{-A}{(z^{-1}-1)(z^{-1}-2)(z^{-1}+3)} = \frac{R_0}{z^{-1}-1} + \frac{R_1}{z^{-1}-2} + \frac{R_2}{z^{-1}+3}$$

$P_0 = 1 \quad P_1 = \frac{1}{2} \quad P_2 = -\frac{1}{3}$

$$R_0 = \frac{-A}{(z^{-1}-2)(z^{-1}+3)}\Big|_{z^{-1}=1} = \frac{A}{4}$$

$$R_1 = \frac{-A}{(z^{-1}-1)(z^{-1}+3)}\Big|_{z^{-1}=2} = -\frac{A}{5}$$

$$R_2 = \frac{-A}{(z^{-1}-1)(z^{-1}-2)}\Big|_{z^{-1}=-3} = -\frac{A}{20}$$

$$y_f(n) = -\frac{A}{4} \cdot +\cancel{(1)^{n+1}} 1_0(n) + \frac{A}{5} \cdot +\left(\frac{1}{2}\right)^{n+1} 1_0(n) + \frac{A}{20} \cdot +\left(-\frac{1}{3}\right)^{n+1} 1_0(n)$$

Es2

$$h(n) = (1+2n)(-1)^n 1_0(n) + \frac{1}{2}\cdot\left(-\frac{1}{2}\right)^n 1_0(n)$$

$$x(n) = \frac{1}{3}\left(-\frac{1}{3}\right)^n 1_0(n) = -\left(-\frac{1}{3}\right)^{n+1} 1_0(n)$$

COND. INIZIALI NULLE

1) BIBO STABILE NO xche $h(n)$ DIVERGE

2) EQ. ALLE DIFFERENZE → CERCHIAMO $H(z)$

3) EVOLUZIONE LIBERA E RISPOSTA FORZATA

$y_e(n) = 0$

$n \geq 0$

SERVE $X(z) = \frac{1}{z^{-1}+3}$

$$-P_0^{n+1} 1_0(n) \longrightarrow \frac{1}{z^{-1}-P_0^{-1}}$$

$$(n+1)P_0^{n+2} 1_0(n) \longrightarrow \frac{1}{(z^{-1}-P_0^{-1})^2}$$

$$h(n) = \underbrace{(1+2n)}_{2(n+1)-1}(-1)^n 1_0(n) + \frac{1}{2}\left(-\frac{1}{2}\right)^n 1_0(n)$$

$$= 2 \cdot (n+1)(-1)^{n+2} 1_0(n) + (-1)^{n+1} 1_0(n) - \left(-\frac{1}{2}\right)^{n+1} 1_0(n)$$

$P_0 = -1 \quad P_0 = -1 \quad P_0 = -\frac{1}{2}$

$$H(z) = 2 \cdot \frac{1}{(z^{-1}+1)^2} - \frac{1}{(z^{-1}+1)} + \frac{1}{z^{-1}+2}$$

$$= \frac{2(z^{-1}+2) - (z^{-1}+1)\overbrace{[(z^{-1}+2) - (z^{-1}+1)]}^{1}}{(z^{-1}+1)^2(z^{-1}+2)}$$

$$= \frac{z^{-1}+3}{(z^{-2}+2z^{-1}+1)(z^{-1}+2)}$$

$$= \frac{z^{-1}+3}{z^{-3}+2z^{-2}+z^{-1}+2z^{-2}+4z^{-1}+2}$$

$$H(z) = \frac{z^{-1}+3}{z^{-3}+4z^{-2}+5z^{-1}+2}$$

$$x(n-1) + 3x(n) = y(n-3) + 4y(n-2) + 5y(n-1) + 2y(n)$$

$$Y_f(z) = H(z)X(z) = \frac{\cancel{z^{-1}+3}}{(z^{-1}+1)^2(z^{-1}+2)} \cdot \frac{1}{\cancel{z^{-1}+3}}$$

$$= \frac{R_0}{(z^{-1}+1)^2} + \frac{R_1}{(z^{-1}+1)} + \frac{R_2}{z^{-1}+2}$$

$$R_0 = Y_f(z)(z^{-1}+1)^2\Big|_{z^{-1}=-1} = \frac{1}{z^{-1}+2}\Big|_{z^{-1}=-1} = 1$$

$$R_1 = \frac{\partial}{\partial z^{-1}}\left(\frac{1}{z^{-1}+2}\right)\Big|_{z^{-1}=-1} = \frac{\partial}{\partial x}\left(\frac{1}{x+2}\right)\Big|_{x=-1}$$

$$= \frac{-1}{(x+2)^2}\Big|_{x=-1} = -1$$

$$R_2 = Y_f(z)(z^{-1}+2)\Big|_{z^{-1}=-2} = \frac{1}{(z^{-1}+1)^2}\Big|_{z^{-1}=-2} = 1$$

$$Y_f(z) = \frac{1}{(z^{-1}+1)^2} - \frac{1}{(z^{-1}+1)} + \frac{1}{(z^{-1}+2)}$$

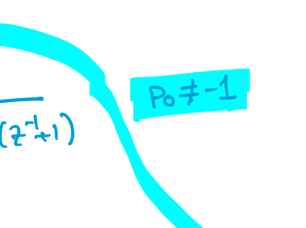
$P_0 = -1 \quad P_0 = -1 \quad P_1 = -\frac{1}{2}$

$$y_f(n) = (n+1)(-1)^{n+2} 1_0(n) + (-1)^{n+1} 1_0(n) - \left(-\frac{1}{2}\right)^{n+1} 1_0(n)$$

NOTA FINALE

$$H(z) = \frac{1}{z^{-1}+1}$$

$$h(n) = (-1)^n 1_0(n)$$



TROVARE $x(n)$ LIMITATO TALE CHE $y_f(n)$ SIA NON LIMITATA

$$X(z) = \frac{1}{z^{-1}-P_0^{-1}} \qquad |P_0| \leq 1$$

$P_0 \neq -1$

$$Y_f(z) = H(z)X(z) = \frac{1}{(z^{-1}-P_0^{-1})(z^{-1}+1)}$$

$$= \frac{R_0}{z^{-1}-P_0^{-1}} + \frac{R_1}{z^{-1}+1}$$

$$y_f(n) = -\underbrace{R_0 P_0^{n+1} 1_0(n)}_{\text{LIMITATO}} - \underbrace{R_1 (-1)^{n+1} 1_0(n)}_{\text{LIMITATO}}$$

$P_0 = -1$

$$Y_f(z) = H(z)X(z) = \frac{1}{(z^{-1}+1)^2}$$

$$y_f(n) = \underbrace{(n+1)(-1)^{n+2}}_{\text{NON LIMITATA}} 1_0(n)$$