Le29

Thursday, 28 May 2026 08:29

Es1 $y''(t) - y'(t) - 6y(t) = x'(t) - 3x(t)$

1) $H(s)$
2) BIBO STABILE
3) RISPOSTA FORZATA CON $x(t) = 1(t)$
4) SE $x(t) = A\cos(\omega_0 t + \varphi_0)$ E LE CONDIZIONI INIZIALI SONO NULLE, TROVARE ω_0 CHE GARANTISCA UN COMPORTAMENTO A REGIME $y(t) = \frac{1}{5}x(t - t_0)$ $t \gg 0$

$$H(s) = \frac{s-3}{s^2 - s - 6} = \frac{\cancel{s-3}\ 1}{\cancel{(s-3)}(s+2)}$$ BIBO STABILE

$$p_{1,2} = \frac{1 \pm \sqrt{1+24}}{2} = \begin{cases} 3 \\ -2 \end{cases}$$

$h(t) = e^{-2t}1(t)$

$$\boxed{\text{NOTA} \quad y'(t) + 2y(t) = x(t) \text{ È MEGLIO}}$$

$$Y_e(s) = \frac{P(s)}{(s-3)(s+2)} = \dots + \frac{R_0}{s-3} + \frac{R_1}{s+2}$$
NON BIBO STABILE

$$Y_f(s) = H(s)X(s) = \frac{1}{s+2} \cdot \frac{1}{s} = \frac{R_0}{s+2} + \frac{R_1}{s}$$

$$R_0 = Y_f(s)(s+2)\Big|_{s=-2} = \frac{1}{s}\Big|_{s=-2} = -\frac{1}{2}$$

$$R_1 = Y_f(s)\, s\Big|_{s=0} = \frac{1}{s+2}\Big|_{s=0} = \frac{1}{2}$$

$$Y_f(s) = \frac{1}{s(s+2)} = \frac{R_0 s + R_1(s+2)}{s(s+2)}$$

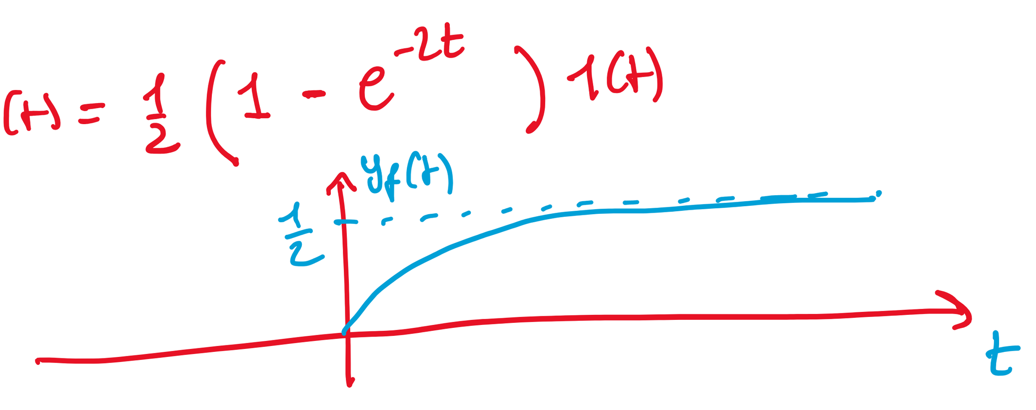
$\Downarrow$

$(R_0 + R_1)s + 2R_1 = 1$

$R_0 + R_1 = 0$
$2R_1 = 1$ $\rightarrow$ $R_1 = 1/2$, $R_0 = -R_1 = -\frac{1}{2}$

$$Y_f(s) = \frac{1}{2}\frac{1}{s} - \frac{1}{2}\frac{1}{s+2}$$

$$y_f(t) = \frac{1}{2}\left(1 - e^{-2t}\right)1(t)$$



$x(t) = A\cos(\omega_0 t + \varphi_0) \rightarrow$ FILTRO REALE $\boxed{h(\cdot)} \rightarrow$ $y(t) = A|H(j\omega_0)|\cos(\omega_0 t + \varphi_0 + \angle H(j\omega_0)) = |H(j\omega_0)|\, x(t - t_0)$ $t \gg 0$

$$H(s) = \frac{1}{s+2}$$

$$H(j\omega_0) = \frac{1}{2 + j\omega_0}$$

$$\frac{1}{5} = |H(j\omega_0)| = \frac{1}{|2 + j\omega_0|}$$

$$|2 + j\omega_0|^2 = 5^2$$

$$4 + \omega_0^2 = 25$$

$$\omega_0^2 = 21$$

$$\omega_0 = \pm\sqrt{21}$$

Es2 TROVARE $X(z)$ PER $x(n) = p_0^n 1_0(n)$ $p_0 \in \mathbb{C}$

$$X(z) = \sum_{n=0}^{+\infty} p_0^n \cancel{1_0(n)} z^{-n} = \sum_{n=0}^{+\infty} (p_0 z^{-1})^n$$

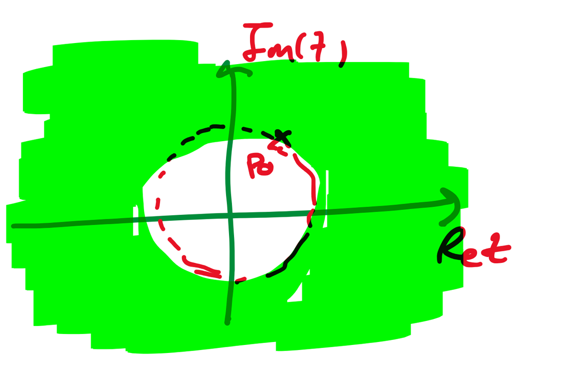
$$= \frac{1}{1 - p_0 z^{-1}}$$

$$= \frac{z}{z - p_0}$$

$|p_0 z^{-1}| < 1$

$\left|\frac{p_0}{z}\right| < 1$

$|p_0| < |z|$



Es3 TROVARE $X(z)$ PER $x(n) = \cos(\theta_0 n) 1_0(n)$

$$x(n) = \frac{1}{2}\underbrace{(e^{j\theta_0})^n}_{p_0 = e^{j\theta_0}} 1_0(n) + \frac{1}{2}\underbrace{(e^{-j\theta_0})^n}_{p_1 = e^{-j\theta_0}} 1_0(n)$$

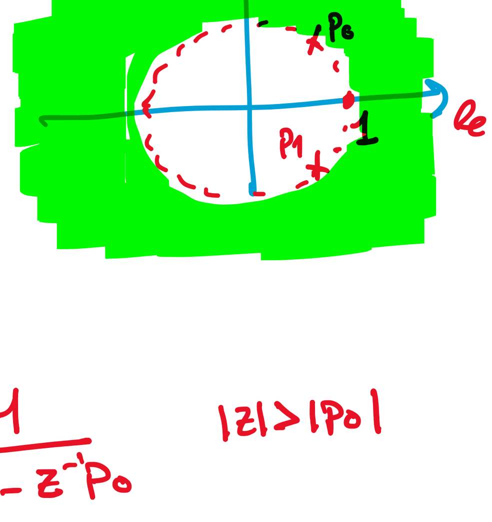
$$X(z) = \frac{1}{2} \cdot \frac{1}{1 - e^{j\theta_0}z^{-1}} + \frac{1}{2} \cdot \frac{1}{1 - e^{-j\theta_0}z^{-1}}$$

$$= \frac{1 - e^{-j\theta_0}z^{-1} + 1 - e^{j\theta_0}z^{-1}}{2(1 - e^{j\theta_0}z^{-1})(1 - e^{-j\theta_0}z^{-1})}$$

$$= \frac{1 - z^{-1}\cancel{\frac{e^{j\theta_0} + e^{-j\theta_0}}{2}}\cos\theta_0}{1 + z^{-2}\cancel{e^{j\theta_0}}\cancel{e^{-j\theta_0}} - z^{-1}\underbrace{\cancel{(e^{j\theta_0} + e^{-j\theta_0})}}_{2\cos\theta_0}}$$

$$= \frac{1 - z^{-1}\cos\theta_0}{1 - 2z^{-1}\cos\theta_0 + z^{-2}}$$

$|z| > |p_0| = 1$
$> |p_1| = 1$



NOTA $p_0^n 1_0(n) \xrightarrow{z} \frac{1}{1 - z^{-1}p_0}$ $|z| > |p_0|$

$$p_0^{n+1} 1_0(n) \xrightarrow{z} \frac{p_0}{1 - z^{-1}p_0} = \frac{1}{p_0^{-1} - z^{-1}} \quad |z| > |p_0|$$

$$\boxed{-p_0^{n+1} 1_0(n) \xrightarrow{z} \frac{1}{z^{-1} - p_0^{-1}} \quad |z| > |p_0|}$$

esempio $\frac{1}{z^{-1} - 3} = X(z)$ $\rightarrow$ IL POLO È $p_0 = \frac{1}{3}$

$$x(n) = -\left(\frac{1}{3}\right)^{n+1} 1_0(n)$$

ES4 TROVARE $X(z)$ PER $x(n) = (n+1)p_0^{n+2} 1_0(n)$

$$x(n) = p_0\left(n p_0^{n+1} 1_0(n) + p_0^{n+1} 1_0(n)\right)$$

$$X(z) = p_0\left(-z \cdot Y'(z) \underbrace{- \frac{1}{z^{-1} - p_0^{-1}}}_{Y(z)}\right)$$

$$Y'(z) = \cancel{+}1 \cdot \frac{+1}{(z^{-1} - p_0^{-1})^2} \cdot -z^{-2}$$

$$X(z) = p_0\left(\cancel{+z} \cdot + \frac{\cancel{z^{2}}\, z^{-1}}{(z^{-1} - p_0^{-1})^2} - \frac{\cancel{1}\, z^{-1} - p_0^{-1}}{(z^{-1} - p_0^{-1})^2}\right)$$

$$= \frac{\cancel{p_0} \cdot \cancel{p_0^{-1}}\ 1}{(z^{-1} - p_0^{-1})^2}$$

$$\boxed{(n+1)p_0^{n+2} 1_0(n) \xrightarrow{z} \frac{1}{(z^{-1} - p_0^{-1})^2}}$$