# Le28

Friday, 22 May 2026 08:34

## ES1 FILTRO RC

$$y(t) + RC\,y'(t) = x(t)$$
$$x(t) = A$$
$$y(0_-) = V_0$$
$$y(t) = ?\quad t>0$$

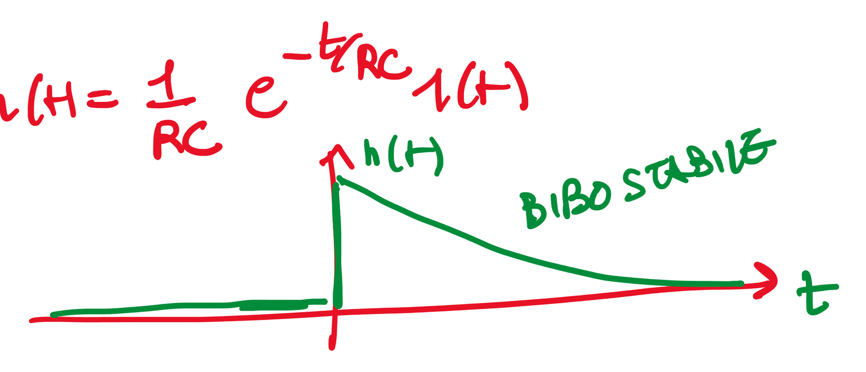
$$y_+(t) + RC\,y'_+(t) = x_+(t) = A\cdot 1(t)$$
$$\downarrow \mathcal{L}^+$$
$$Y(s) + RC\big(sY(s) - \underbrace{y(0_-)}_{V_0}\big) = X(s) = \frac{A}{s}$$

$$\cancel{(1+sRC)}\,Y(s) - \cancel{V_0RC} = \frac{X(s)\cdot 1}{1+sRC} + \frac{V_0RC}{1+sRC}$$

$$Y(s) = \overbrace{\underbrace{\frac{1}{1+sRC}}_{H(s)}X(s)}^{\text{RISPOSTA FORZATA}} + \underbrace{\frac{V_0RC}{1+sRC}}_{\text{EVOLUZIONE LIBERA}}$$

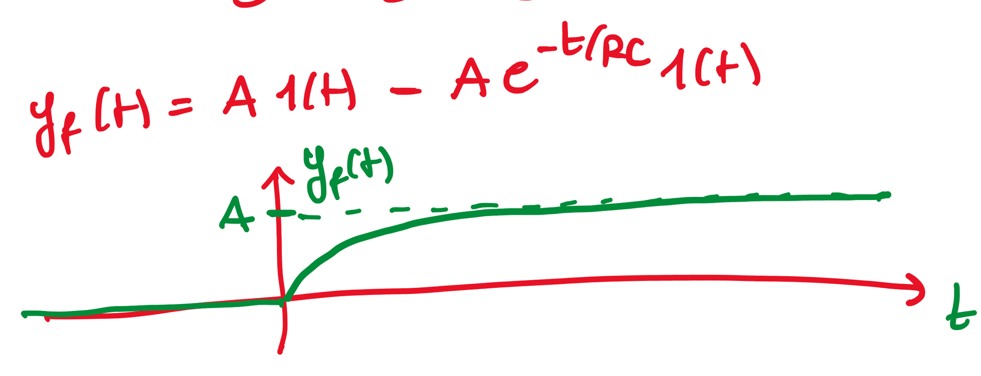
$$H(s) = \frac{1}{1+sRC} = \frac{1/RC}{s+1/RC}$$

$$h(t) = \frac{1}{RC}e^{-t/RC}1(t)$$



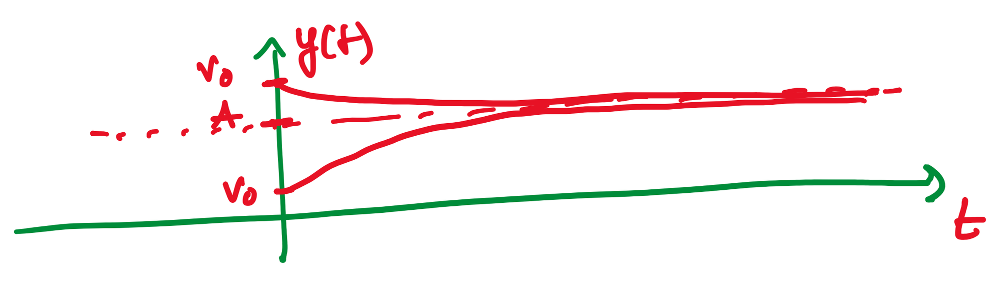

$$Y_f(s) = H(s)X(s) = \frac{A/RC}{s(s+1/RC)}$$
$$= \frac{R_0}{s} + \frac{R_1}{s+1/RC}$$
$$R_0 = Y_f(s)\,s\big|_{s=0} = \frac{A/RC}{s+1/RC}\bigg|_{s=0} = A$$
$$R_1 = Y_f(s)\left(s+\tfrac{1}{RC}\right)\bigg|_{s=-\frac{1}{RC}} = \frac{A/RC}{s}\bigg|_{s=-\frac{1}{RC}} = -A$$

$$Y_f(s) = \frac{A}{s} - \frac{A}{s+1/RC}$$
$$y_f(t) = A\,1(t) - A e^{-t/RC}1(t)$$




$$Y_e(s) = \frac{V_0RC}{1+sRC} = \frac{V_0}{s+\frac{1}{RC}}$$
$$y_e(t) = V_0 e^{-t/RC}1(t)$$

$$y(t) = y_f(t) + y_e(t)$$
$$= A\,1(t) + (V_0-A)e^{-t/RC}1(t)$$




## ES2 SISTEMA MASSA/MOLLA

$$x(t) = K\,y(t) + m\,y''(t)$$
$$x(t) = F_0\cos(\omega_0 t)$$
$$y(0_-) = y_0$$
$$y'(0_-) = v_0$$
$$y(t) = ?\quad t>0$$

$$x_+(t) = K\,y_+(t) + m\,y''_+(t) \qquad x_+(t) = F_0\cos(\omega_0 t)\,1(t)$$
$$\downarrow \mathcal{L}^+$$
$$X(s) = K\,Y(s) + m\big(s^2Y(s) - s\,\underbrace{y(0_-)}_{y_0} - \underbrace{y'(0_-)}_{v_0}\big)$$

$$X(s) = (K+ms^2)Y(s) - m(v_0 + s y_0)$$

$$Y(s) = \frac{1/m}{\frac{K}{m}+\cancel{m}s^2}X(s) + \frac{\cancel{m}(v_0+sy_0)}{\frac{K}{m}+\cancel{m}s^2}$$

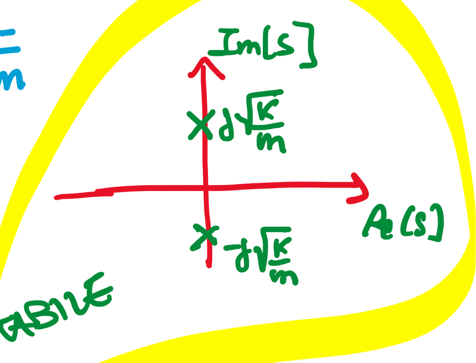
$$= \underbrace{\boxed{\frac{1/m}{s^2+\frac{K}{m}}}}_{H(s)}X(s) + \underbrace{\boxed{\frac{v_0+sy_0}{s^2+\frac{K}{m}}}}_{Y_e(s)}$$

$$x_+(t) = F_0\cos(\omega_0 t)1(t)$$
$$\downarrow \mathcal{L}$$
$$X(s) = \frac{F_0 s}{s^2+\omega_0^2}$$

$$\boxed{\begin{aligned} &\cos(\omega_0 t)1(t) \xrightarrow{\mathcal{L}} \frac{s}{s^2+\omega_0^2}\\ &\sin(\omega_0 t)1(t) \xrightarrow{\mathcal{L}} \frac{\omega_0}{s^2+\omega_0^2}\end{aligned}}$$

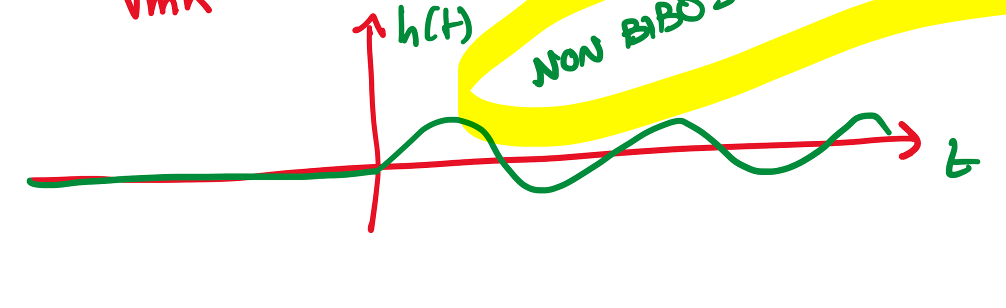
$$H(s) = \frac{1/m}{s^2+\frac{K}{m}} = \frac{1/m}{\left(s-j\sqrt{\frac{K}{m}}\right)\left(s+j\sqrt{\frac{K}{m}}\right)}$$
$$\left(= \frac{R_0}{s-j\sqrt{\frac{K}{m}}} + \frac{R_1}{s+j\sqrt{\frac{K}{m}}}\right)$$

$$= \underbrace{\frac{\sqrt{\frac{K}{m}}}{s^2+\frac{K}{m}}}_{\sin\sqrt{\frac{K}{m}}t}\cdot \underbrace{\frac{1}{m}\sqrt{\frac{m}{K}}}_{\frac{1}{\sqrt{Km}}}$$



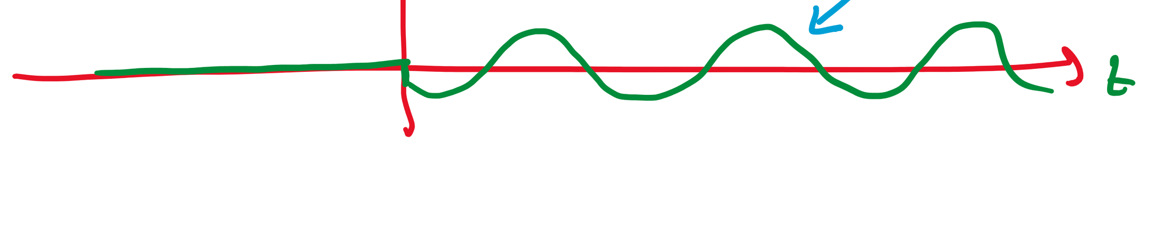

$$h(t) = \frac{1}{\sqrt{mK}}\sin\left(\sqrt{\frac{K}{m}}t\right)1(t)$$

NON BIBO STABILE




$$Y_e(s) = \frac{v_0+sy_0}{s^2+\frac{K}{m}} = \frac{v_0}{\sqrt{\frac{K}{m}}}\cdot\frac{\sqrt{\frac{K}{m}}}{s^2+\frac{K}{m}} + y_0\cdot\frac{s}{s^2+\frac{K}{m}}$$

$$y_e(t) = \left[\sqrt{\frac{m}{K}}\,v_0\sin\left(\sqrt{\frac{K}{m}}t\right) + y_0\cos\left(\sqrt{\frac{K}{m}}t\right)\right]1(t)$$




$$Y_f(s) = H(s)X(s)$$
$$= \frac{1/m}{s^2+\frac{K}{m}}\cdot\frac{F_0 s}{s^2+\omega_0^2}$$

poli $\pm j\omega_0$ $\pm j\sqrt{\frac{K}{m}}$ $\omega_0 \neq \sqrt{\frac{K}{m}}$

$$\frac{1}{s^2+\frac{K}{m}}\cdot\frac{1}{s^2+\omega_0^2} \xrightarrow{s^2=x} \frac{1}{\left(x+\frac{K}{m}\right)(x+\omega_0^2)} = \frac{R_0}{x+\frac{K}{m}} + \frac{R_1}{x+\omega_0^2}$$

$$\xrightarrow{x=s^2}\quad = \frac{R_0}{s^2+\frac{K}{m}} + \frac{R_1}{s^2+\omega_0^2}$$

$$R_0 = \frac{1}{x+\omega_0^2}\bigg|_{x=-\frac{K}{m}} = \frac{1}{\omega_0^2-\frac{K}{m}}$$
$$R_1 = \frac{1}{x+\frac{K}{m}}\bigg|_{x=-\omega_0^2} = \frac{1}{\frac{K}{m}-\omega_0^2} = -R_0$$

$$Y_f(s) = \frac{F_0}{\cancel{m}}\cancel{s}\cdot\cancel{R_0}\left(\frac{\cancel{1}s}{s^2+\frac{K}{m}} - \frac{\cancel{1}s}{s^2+\omega_0^2}\right)\frac{1}{m\omega_0^2 - \cancel{\frac{K}{m}}}$$

$$\boxed{y_f(t) = \frac{F_0}{m\omega_0^2-K}\left(\cos\left(\sqrt{\frac{K}{m}}t\right) - \cos(\omega_0 t)\right)1(t) \qquad \omega_0 \neq \sqrt{\frac{K}{m}}}$$

**NOTA** $x_+ * h(t)$