NOTA

$$H(s) = \frac{R_1}{(s-p_1)} + \sum_k \frac{R_k}{(s-p_k)} \qquad p_k \neq p_1$$

$$H(s)(s-p_1)\Big|_{s=p_1} = \frac{R_1}{(s-p_1)}(s-p_1) + \sum_k \frac{R_k}{(s-p_k)}(s-p_1) = R_1$$

($(s-p_1) = 0$, $(p_1-p_k) \neq 0$)

ES1 TROVARE $x(t)$ CAUSALE CON $X(s) = \frac{s-3}{s+2}$

FRAZIONE PROPRIA O IMPROPRIA?

$b(s)$: $a(s)$

$$\begin{array}{l} s-3 \; : \; s+2 = 1 \quad q(s) \text{ QUOZIENTE} \\ -s-2 \\ \hline 0 \; -5 \quad r(s) \text{ RESTO} \end{array}$$

$$b(s) = a(s)\,q(s) + r(s)$$

$$X(s) = \frac{b(s)}{a(s)} = \frac{a(s)q(s) + r(s)}{a(s)} = q(s) + \frac{r(s)}{a(s)}$$

$$= 1 - \frac{5}{s+2} = \frac{s+2-5}{s+2} = \frac{s-3}{s+2} \checkmark$$

$s_0 = -2$

$\delta(t) \rightarrow 1$

$e^{s_0 t}1(t) \rightarrow \frac{1}{s-s_0}$

$$x(t) = \delta(t) - 5e^{-2t}1(t)$$

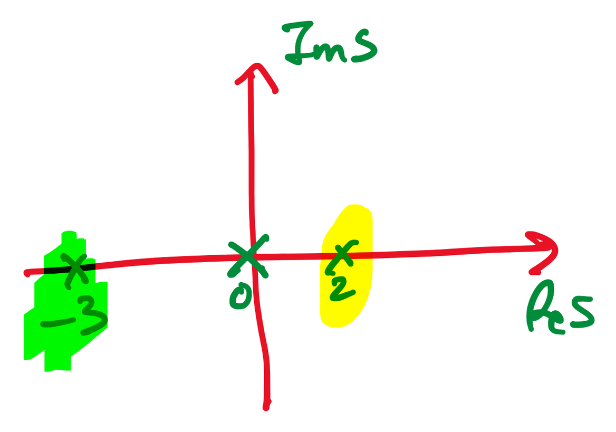
ES2 TROVARE L'ANTITRASFORMATA CAUSALE DI $X(s) = \frac{1}{s^3+s^2-6s}$ PROPRIA

$$X(s) = \frac{1}{s(s^2+s-6)}$$

$\hookrightarrow p_{1,2} = \frac{-1 \pm \sqrt{1+24}}{2}$ (5) $= 2,\ -3$

$$X(s) = \frac{1}{s(s-2)(s+3)}$$

$$= \frac{R_0}{s} + \frac{R_1}{s-2} + \frac{R_2}{s+3}$$



$$R_0 = X(s)\cdot s\Big|_{s=0} = \frac{1}{(s-2)(s+3)}\Big|_{s=0} = -\frac{1}{6}$$

$$R_1 = X(s)\cdot(s-2)\Big|_{s=2} = \frac{1}{s(s+3)}\Big|_{s=2} = \frac{1}{10}$$

$$R_2 = X(s)(s+3)\Big|_{s=-3} = \frac{1}{s(s-2)}\Big|_{s=-3} = \frac{1}{15}$$

$$X(s) = -\frac{1}{6}\cdot\frac{1}{s} + \frac{1}{10}\cdot\frac{1}{s-2} + \frac{1}{15}\cdot\frac{1}{s+3}$$

$\mathcal{L}^{-1}\downarrow$

$$x(t) = -\frac{1}{6}\cdot 1(t) + \frac{1}{10}\cdot e^{2t}1(t) + \frac{1}{15}\cdot e^{-3t}1(t)$$

NOTA

$$X(s) = \frac{R_0}{s-p_0} + \frac{R_1}{(s-p_0)^2} + \frac{R_2}{(s-p_0)^3} + \sum_k \frac{R_k}{(s-p_k)^{m_k}} \qquad p_k \neq p_0$$

POLO DI MOLTEPLICITÀ 3

$$X(s)(s-p_0)^3\Big|_{s=p_0} = R_0(s-p_0)^2 + R_1(s-p_0) + R_2 + \sum_k R_k \frac{(s-p_0)^3}{(s-p_k)^{m_k}}$$

($(s-p_0)^3 = 0$, $(p_0-p_k)^{m_k} \neq 0$)

$= R_2$

$$\frac{d}{ds}\left(X(s)(s-p_0)^3\right)\Big|_{s=p_0} = 2R_0(s-p_0) + R_1 + 0 + \sum_k R_k \frac{3(s-p_0)^2}{(s-p_k)^{m_k}} - \sum_k R_k \frac{(s-p_0)^3 m_k}{(s-p_k)^{m_k+1}}$$

$= R_1$

$$\frac{d^2}{ds^2}\left(X(s)(s-p_0)^3\right)\Big|_{s=p_0} = 2R_0 + 0 + 0 + \sum_k R_k \frac{3\cdot 2(s-p_0)}{(s-p_k)^{m_k}} - R_k\frac{3(s-p_0)^2 m_k}{(s-p_k)^{m_k+1}} - R_k\frac{3(s-p_0)^2 m_k}{(s-p_k)^{m_k+1}} + R_k\frac{(s-p_0)^3 m_k(m_k+1)}{(s-p_k)^{m_k+2}}$$

$= 2R_0$

ES3 TROVARE L'ANTITRASFORMATA CAUSALE DI

$$X(s) = \frac{4s-1}{2s^2(s-1)} = \frac{2s-1/2}{s^2(s-1)}$$

POLO 0 CON MOLTEPLICITÀ 2

POLO 1 CON MOLTEPLICITÀ 1

$$X(s) = \frac{R_0}{s} + \frac{R_1}{s^2} + \frac{R_2}{s-1}$$

$$R_2 = X(s)(s-1)\Big|_{s=1} = \frac{2s-1/2}{s^2}\Big|_{s=1} = \frac{3}{2}$$

$$R_1 = X(s)\,s^2\Big|_{s=0} = \frac{2s-1/2}{s-1}\Big|_{s=0} = \frac{1}{2}$$

$$R_0 = \frac{d}{ds}\left(X(s)s^2\right)\Big|_{s=0} = \frac{d}{ds}\left(\frac{2s-1/2}{s-1}\right)\Big|_{s=0}$$

$$= \frac{2}{s-1} - \frac{(2s-1/2)}{(s-1)^2}\Big|_{s=0} = -2 + \frac{1}{2} = -\frac{3}{2}$$

$$X(s) = -\frac{3}{2}\cdot\frac{1}{s} + \frac{1}{2}\frac{1}{s^2} + \frac{3}{2}\frac{1}{s-1}$$

$\mathcal{L}^{-1}\downarrow$

$$x(t) = -\frac{3}{2}\cdot 1(t) + \frac{1}{2}\cdot t\cdot 1(t) + \frac{3}{2}\cdot e^{t}1(t)$$