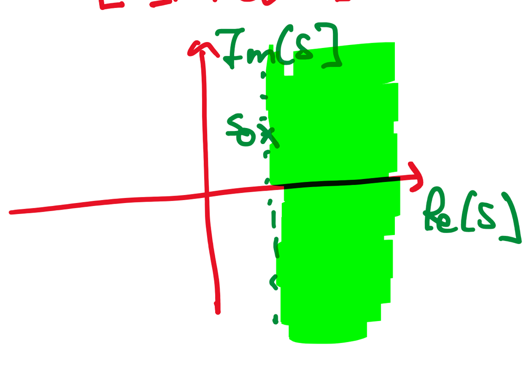
Le26

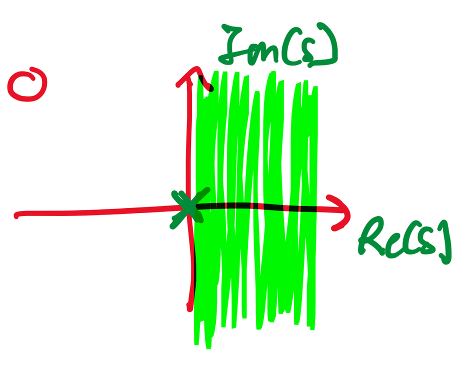
Friday, 15 May 2026 08:29

NOTA $e^{s_0 t} 1(t) \xrightarrow{\mathcal{L}} \frac{1}{s-s_0}$ $Re[s] > Re[s_0]$



ES1 TROVARE $X(s)$ PER $x(t) = 1(t) = e^{s_0 t} 1(t)$ CON $s_0 = 0$

$1(t) \xrightarrow{\mathcal{L}} \frac{1}{s}$ $Re[s] > 0$

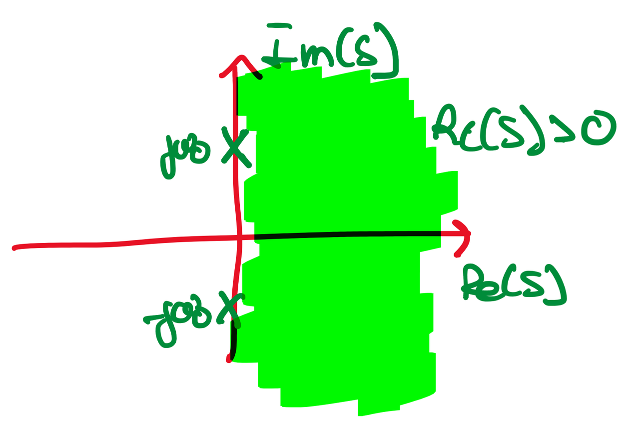


ES2 TROVARE $X(s)$ PER $x(t) = \cos(\omega_0 t)\, 1(t)$

$$x(t) = \frac{1}{2} \underbrace{e^{j\omega_0 t} 1(t)}_{\mathcal{L},\ s_0 = j\omega_0} + \frac{1}{2} \underbrace{e^{-j\omega_0 t} 1(t)}_{\mathcal{L},\ s_0 = -j\omega_0}$$

$Re[s] > Re[j\omega_0] = 0$ $\quad$ $Re[s] > Re[-j\omega_0] = 0$

$$X(s) = \frac{1}{2} \frac{1}{s - j\omega_0} + \frac{1}{2} \frac{1}{s + j\omega_0}$$



$$X(s) = \frac{s + \cancel{j\omega_0} + s - \cancel{j\omega_0}}{\cancel{2}(s - j\omega_0)(s + j\omega_0)}$$

$$= \frac{s}{s^2 - (j\omega_0)^2} = \frac{s}{s^2 + \omega_0^2}$$

x CASA $\boxed{\sin(\omega_0 t) \xrightarrow{\mathcal{L}} \frac{\omega_0}{s^2 + \omega_0^2}}$

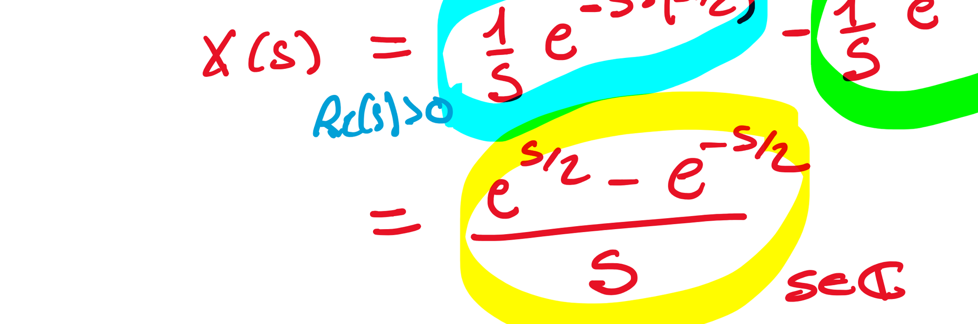
ES3 TROVARE $X(s)$ PER $x(t) = \mathrm{rect}(t)$

$$X(s) = \int_{-\frac{1}{2}}^{\frac{1}{2}} \cancel{\mathrm{rect}(t)}\, e^{-st} dt = \left. \frac{e^{-st}}{-s} \right]_{-1/2}^{1/2}$$

$\Gamma = \mathbb{C}$

$$= \frac{-e^{-\frac{1}{2}s} + e^{+(\frac{1}{2})s}}{+s} = \begin{cases} \dfrac{e^{s/2} - e^{-s/2}}{s} & s \neq 0 \\ \int_{-1/2}^{1/2} 1\, dt = 1 & s = 0 \end{cases}$$

OSSERVAZIONE $\mathrm{rect}(t) = 1(t + 1/2) - 1(t - 1/2)$



$$\mathrm{rect}(t) = 1(t - (-1/2)) - 1(t - 1/2)$$

$$X(s) = \frac{1}{s} e^{-s \cdot (-1/2)} - \frac{1}{s} e^{-s \cdot 1/2} \quad Re[s] > 0$$

$Re(s) > 0$

$$= \frac{e^{s/2} - e^{-s/2}}{s} \quad s \in \mathbb{C}$$

ES4 TROVARE $X(s)$ PER $x(t) = \delta(t)$

$$X(s) = \int_{-\infty}^{+\infty} \delta(t)\, e^{-st} dt = e^{-st} \Big|_{t=0} = 1$$

$s \in \mathbb{C}$

ES5 TROVARE $X(s)$ PER $x(t) = \delta(t - t_0)$

regola di traslazione

$$X(s) = 1 \cdot e^{-s t_0} = e^{-s t_0} \quad s \in \mathbb{C}$$

ES6 TROVARE $X(s)$ PER $x(t) = \delta'(t)$

regola di derivazione NEL TEMPO

$\delta'(t) \xrightarrow{\mathcal{L}} 1 \cdot s = s$

$\delta''(t) \xrightarrow{\mathcal{L}} s^2$

$\delta'''(t) \xrightarrow{\mathcal{L}} s^3$

$\boxed{\delta^{(k)}(t) \xrightarrow{\mathcal{L}} s^k}$ $\quad s \in \mathbb{C}$

ES7 TROVARE $X(s)$ PER $x(t) = t^k e^{s_0 t} 1(t)$

$e^{s_0 t} 1(t) \xrightarrow{\mathcal{L}} \frac{1}{s - s_0}$

$t \cdot e^{s_0 t} 1(t) \xrightarrow{\mathcal{L}} -1 \cdot \frac{\partial}{\partial s}\left(\frac{1}{s - s_0}\right) = +1 \cdot +1 \cdot \frac{1}{(s-s_0)^2}$

REGOLA DI DERIVAZIONE IN S

$= \frac{1}{(s-s_0)^2}$

$t^2 e^{s_0 t} 1(t) \xrightarrow{\mathcal{L}} -1 \cdot -2 \cdot \frac{1}{(s-s_0)^3} = \frac{2}{(s-s_0)^3}$

$t^3 e^{s_0 t} 1(t) \xrightarrow{\mathcal{L}} -1 \cdot -3 \cdot \frac{2}{(s-s_0)^4} = \frac{2 \cdot 3}{(s-s_0)^4}$

$t^4 e^{s_0 t} 1(t) \xrightarrow{\mathcal{L}} -1 \cdot -4 \cdot \frac{2 \cdot 3}{(s-s_0)^5} = \frac{2 \cdot 3 \cdot 4}{(s-s_0)^5}$

$$\boxed{t^k e^{s_0 t} 1(t) \xrightarrow{\mathcal{L}} \frac{k!}{(s-s_0)^{k+1}}}$$

x INDUZIONE

$t^{k+1} e^{s_0 t} 1(t) \xrightarrow{\mathcal{L}} -1 \cdot -(k+1) \cdot \frac{k!}{(s-s_0)^{k+2}}$

$= \frac{(k+1)!}{(s-s_0)^{k+2}}$ ✓

$$\boxed{\frac{t^k}{k!} e^{s_0 t} 1(t) \xrightarrow{\mathcal{L}} \frac{1}{(s-s_0)^{k+1}}}$$

$Re(s) > Re[s_0]$

NOTA PONIAMO $s_0 = 0$

$$\boxed{\frac{t^k}{k!} 1(t) \xrightarrow{\mathcal{L}} \frac{1}{s^{k+1}}}$$

CONVOLUZIONE K+1 GRADINI

PRODOTTO DI K+1 GRADINI

$1(t)$

$$1 * 1(t) = \int_{0}^{t} \cancel{1(u)}\, \cancel{1(t-u)}\, du \qquad t > 0$$

$$= t \cdot 1(t)$$

REGOLA DI DERIVAZIONE $\boxed{sX(s) - x(0_-)}$