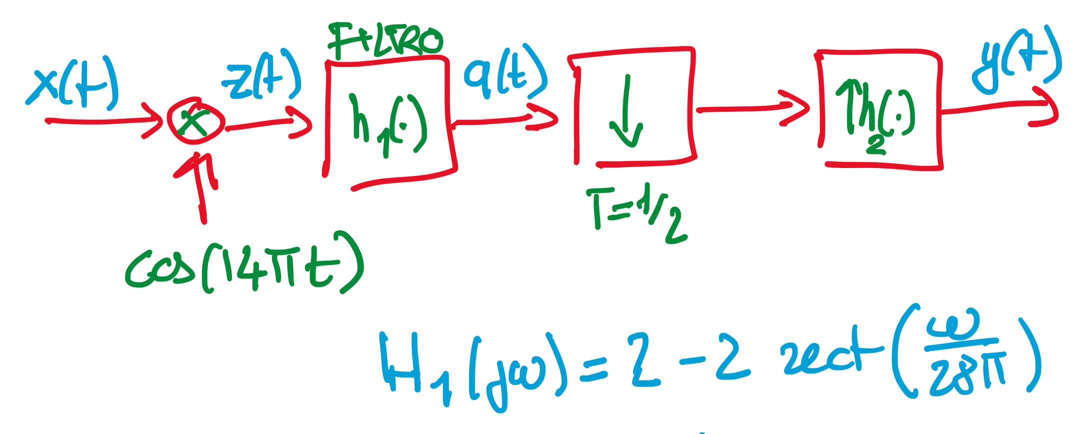
Le25

Thursday, 14 May 2026 08:27

Es 1

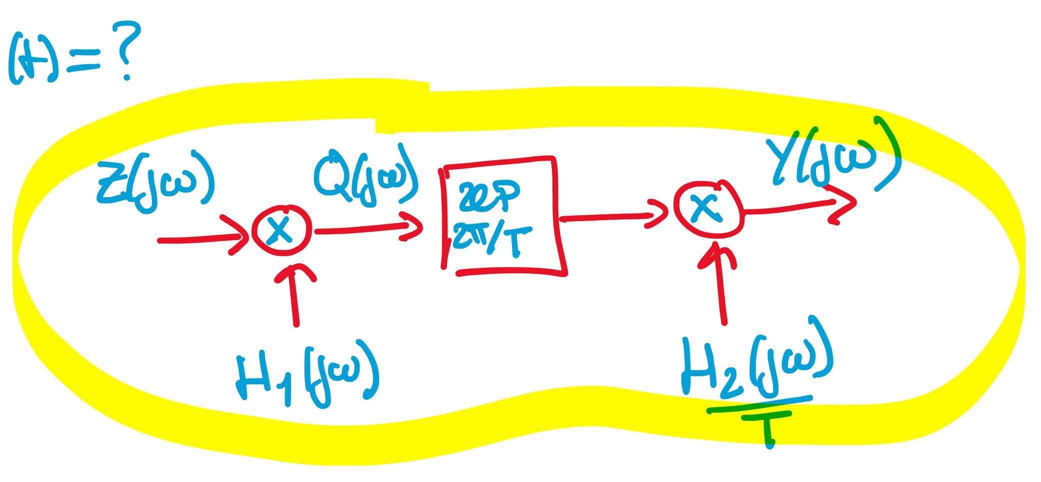


$$H_1(j\omega) = 2 - 2\,\mathrm{rect}\left(\frac{\omega}{28\pi}\right)$$

$$H_2(j\omega) = \frac{1}{2}\,\mathrm{rect}\left(\frac{\omega}{4\pi}\right)$$

$$X(j\omega) = \mathrm{rect}\left(\frac{\omega}{4\pi}\right)\cdot\left(1 - \mathrm{triang}\left(\frac{\omega}{2\pi}\right)\right)$$

$y(t) = ?$

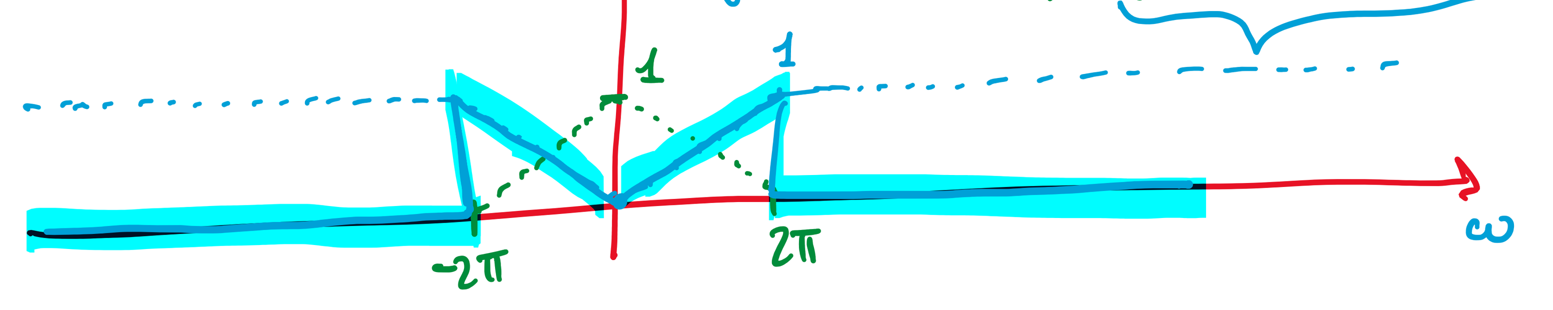


$$z(t) = x(t)\cos(\omega_0 t) \qquad \omega_0 = 14\pi$$

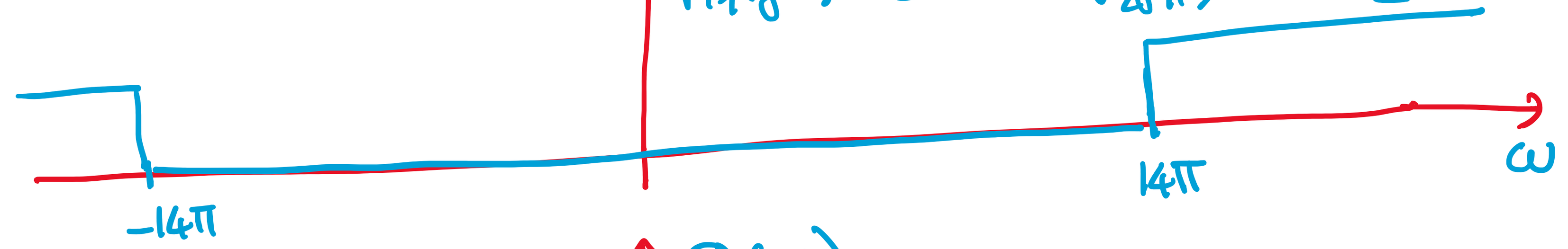
$$= \frac{1}{2}x(t)e^{+j\omega_0 t} + \frac{1}{2}x(t)e^{-j\omega_0 t}$$

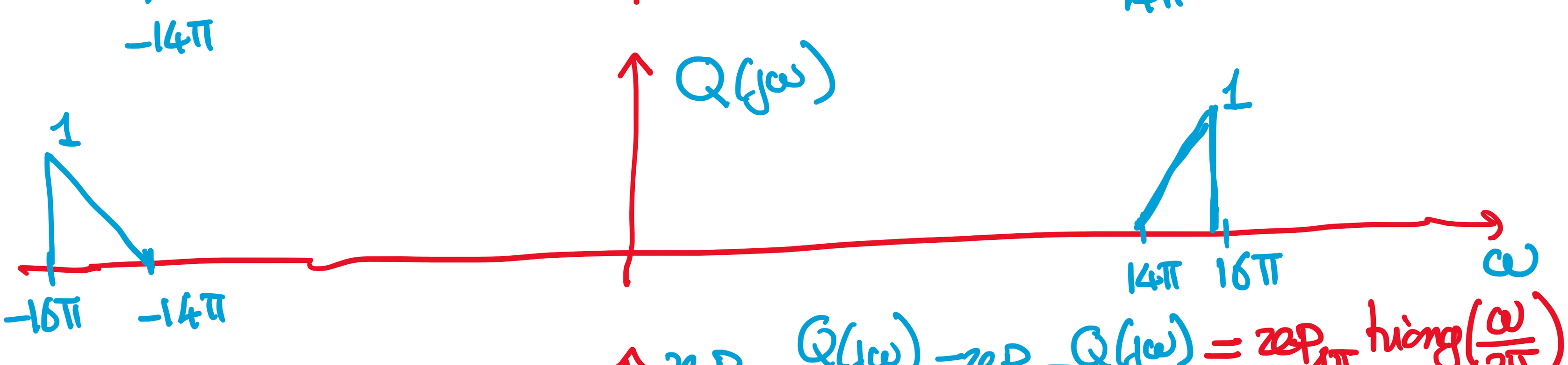
$$Z(j\omega) = \frac{1}{2}X(j(\omega-\omega_0)) + \frac{1}{2}X(j(\omega+\omega_0))$$

$$X(j\omega) = \mathrm{rect}\left(\frac{\omega}{4\pi}\right)\cdot\underbrace{\left(1 - \mathrm{triang}\left(\frac{\omega}{2\pi}\right)\right)}$$

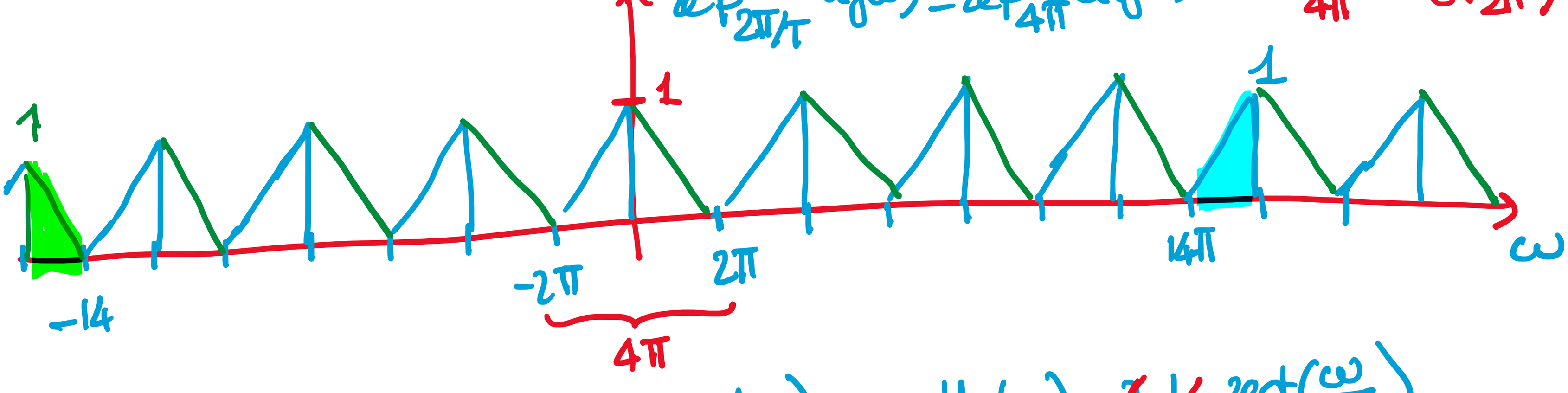


$$H_1(j\omega) = 2 - 2\,\mathrm{rect}\left(\frac{\omega}{28\pi}\right)$$

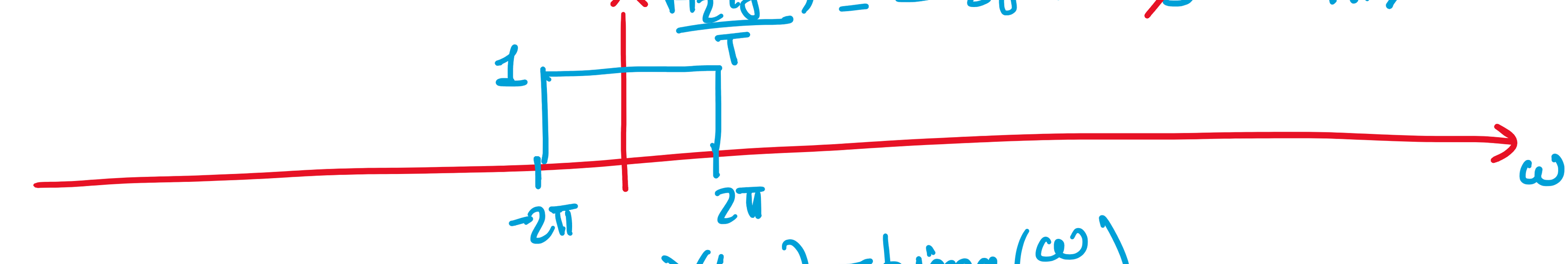




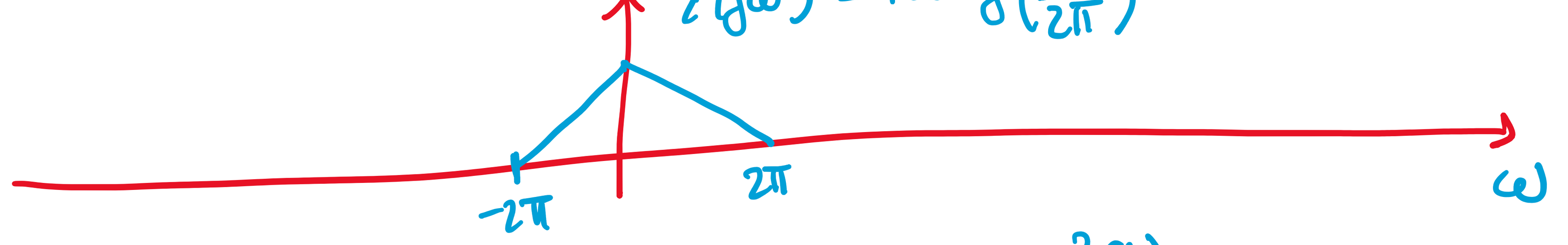
$$\mathrm{rep}_{2\pi/T}\,Q(j\omega) = \mathrm{rep}_{4\pi}\,Q(j\omega) = \mathrm{rep}_{4\pi}\,\mathrm{triang}\left(\frac{\omega}{2\pi}\right)$$



$$\frac{H_2(j\omega)}{T} = 2H_2(j\omega) = \cancel{2}\cdot\cancel{\tfrac{1}{2}}\cdot\mathrm{rect}\left(\frac{\omega}{4\pi}\right)$$



$$Y(j\omega) = \mathrm{triang}\left(\frac{\omega}{2\pi}\right)$$



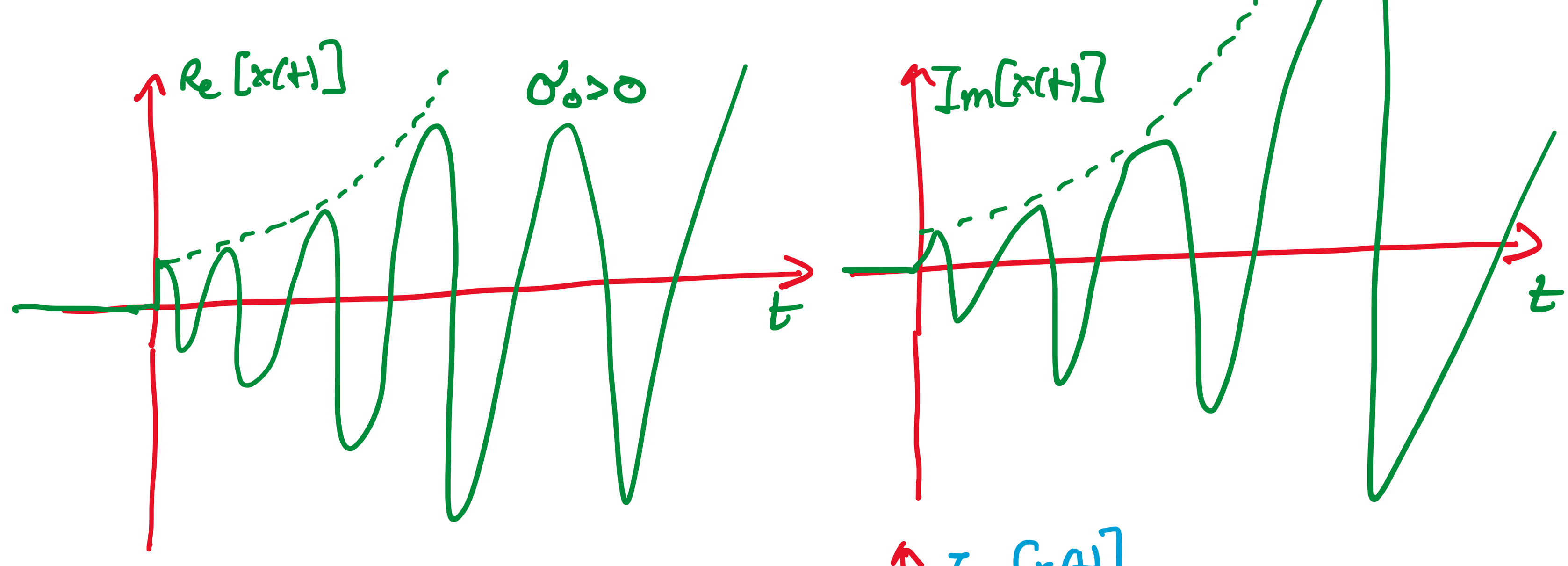
$$y(t) = \mathrm{sinc}^2(t)$$

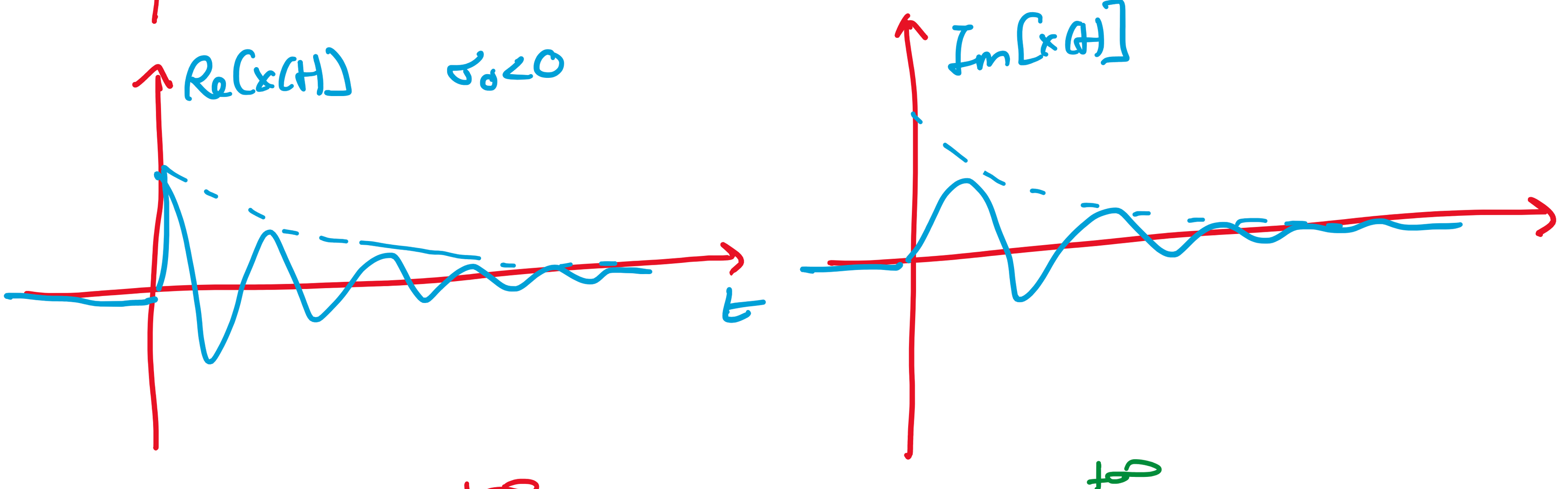
Es2 CALCOLARE $X(s)$ PER $x(t) = e^{s_0 t}1(t)$, $s_0 \in \mathbb{C}$

$$s_0 = \sigma_0 + j\omega_0$$

$$x(t) = e^{\sigma_0 t}e^{j\omega_0 t}1(t)$$

$$= e^{\sigma_0 t}\cos(\omega_0 t)1(t) + j\,e^{\sigma_0 t}\sin(\omega_0 t)1(t)$$





$$X(s) = \int_{\cancel{-\infty}\,0}^{+\infty} e^{s_0 t}\cancel{1(t)}\cdot e^{-st}\,dt = \int_0^{+\infty} e^{(s_0-s)t}\,dt$$

$$= \frac{e^{(s_0-s)t}}{s_0-s}\Bigg|_0^{+\infty} = \frac{\lim_{t\to\infty} e^{(s_0-s)t} - 1}{s_0-s}$$

$$s_0 - s = a + jb \qquad a = Re[s_0-s] \quad b = Im[s_0-s]$$

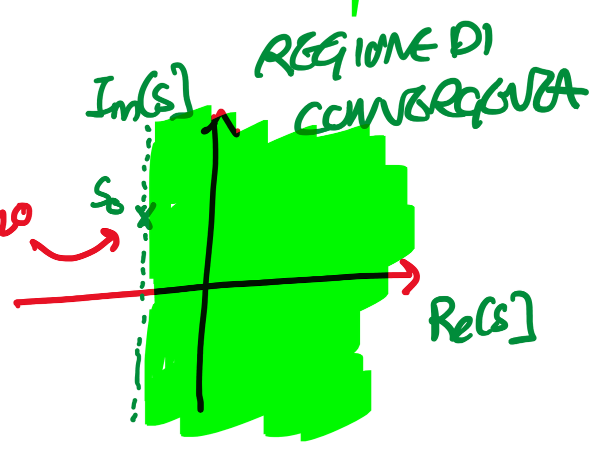
$$\lim_{t\to\infty} e^{at}e^{jbt}$$

$$X(s) = \begin{cases} \text{INDEFINITO} & Re[s_0-s] \geq 0 \\ \frac{-1}{s_0-s} = \frac{1}{s-s_0} & Re[s_0-s] < 0 \end{cases}$$

$$x(t) = e^{s_0 t}1(t) \xrightarrow{\mathcal{L}} X(s) = \frac{1}{s-s_0} \qquad \Gamma = \{s \mid Re[s] > Re[s_0]\}$$

VALORE ANALITICO

REGIONE DI CONVERGENZA



Es 3 TROVARE $X(s)$ PER $x(t) = -e^{s_0 t}1(-t)$

$$X(s) = \int_{-\infty}^{\cancel{+\infty}\,0} -e^{s_0 t}\cancel{1(-t)}\cdot e^{-st}\,dt = -\int_{-\infty}^{0} e^{(s_0-s)t}\,dt$$

$$= \frac{-e^{(s_0-s)t}}{s_0-s}\Bigg|_{-\infty}^{0} = \frac{e^{(s_0-s)t}}{s-s_0}\Bigg|_{-\infty}^{0}$$

$$= \frac{1 - \lim_{t\to-\infty} e^{(s_0-s)t}}{s-s_0}$$

$$X(s) = \begin{cases} \text{INDEFINITO} & Re[s_0-s] \leq 0 \\ \frac{1}{s-s_0} & Re[s_0-s] > 0 \end{cases}$$

$$x(t) = -e^{s_0 t}1(-t) \xrightarrow{\mathcal{L}} X(s) = \frac{1}{s-s_0} \qquad \Gamma = \{s \mid Re[s] < Re[s_0]\}$$