

ES1 TROVARE $S(e^{j\omega})$ PER $\Delta(n) = \delta(n)$

$$S(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} \delta(n) e^{-jn\omega} = e^{-jn\omega} \Big|_{n=0} = 1$$

ES2 $\Delta(n) = \delta(n-n_0)$

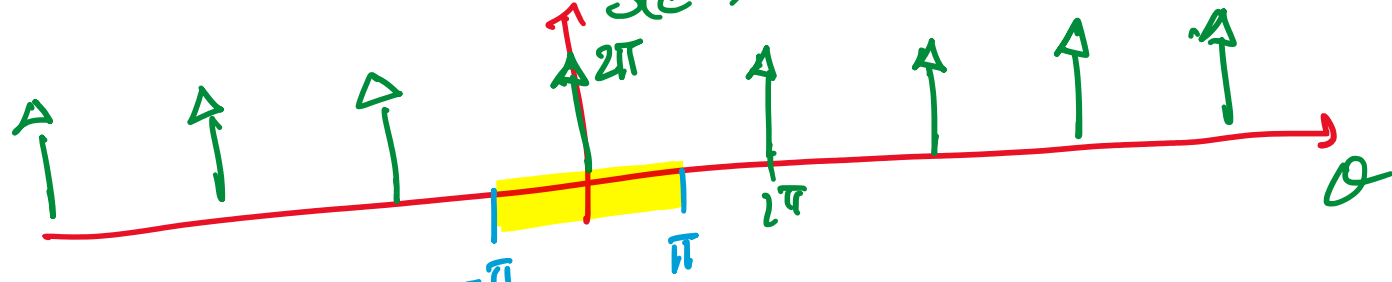
$$S(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} \delta(n-n_0) e^{-jn\omega} = e^{-jn_0\omega} \Big|_{n=n_0} = e^{-jn_0\omega}$$

ES3 $\Delta(n) = 1$

$$S(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} 1 \cdot e^{-jn\omega} = ?$$

$$\Delta(n) = \frac{1}{2\pi} \int_0^{2\pi} S(e^{j\omega}) e^{jn\omega} d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi}^{2\pi} 2\pi \text{comb}(\frac{\omega}{2\pi}) e^{jn\omega} d\omega = e^{jn\omega} \Big|_{\omega=0} = 1$$



$$\Delta(n) = 1 \xrightarrow{\text{if}} S(e^{j\omega}) = 2\pi \text{comb}(\frac{\omega}{2\pi})$$

ES4 $\Delta(n) = \cos(\theta_0 n + \theta_1)$

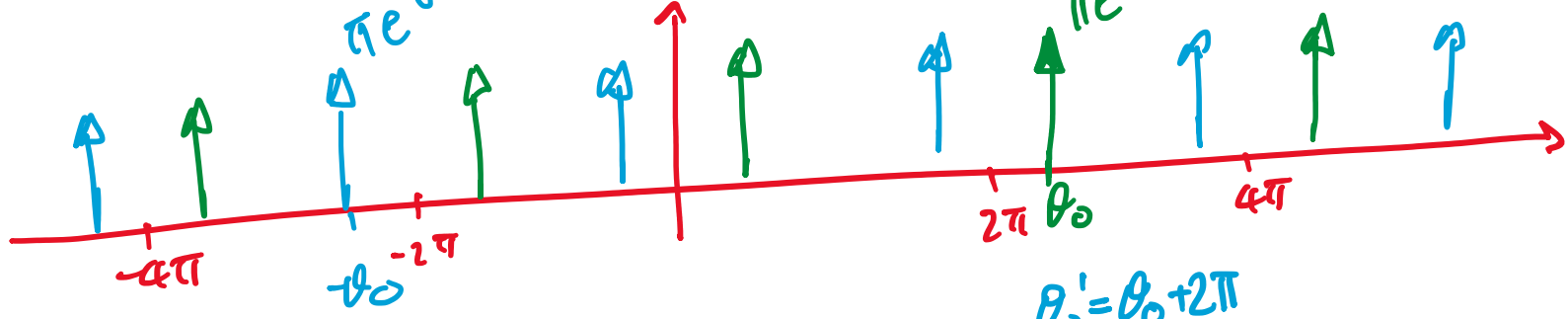
$$\Downarrow = \frac{e^{j\theta_1}}{2} \cdot e^{j\theta_0 n} \cdot 1 + \frac{e^{-j\theta_1}}{2} \cdot e^{-j\theta_0 n} \cdot 1$$

$$S(e^{j\omega}) = \frac{e^{j\theta_1}}{2} \cdot 2\pi \text{comb}(\frac{\omega-\theta_0}{2\pi}) + \frac{e^{-j\theta_1}}{2} \cdot 2\pi \text{comb}(\frac{\omega+\theta_0}{2\pi})$$

$$\cos(\theta_0 n + \theta_1) = \pi e^{j\theta_1} \text{comb}(\frac{\omega-\theta_0}{2\pi}) + \pi e^{-j\theta_1} \text{comb}(\frac{\omega+\theta_0}{2\pi})$$



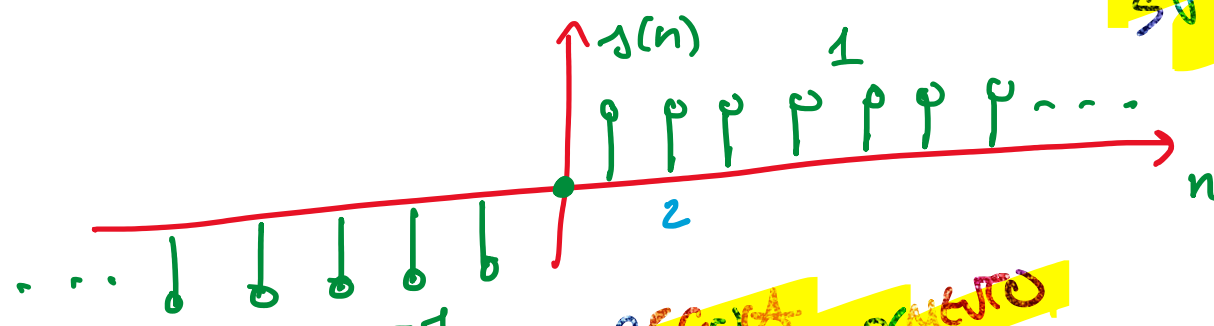
SIMMETRIA REALIUMMA



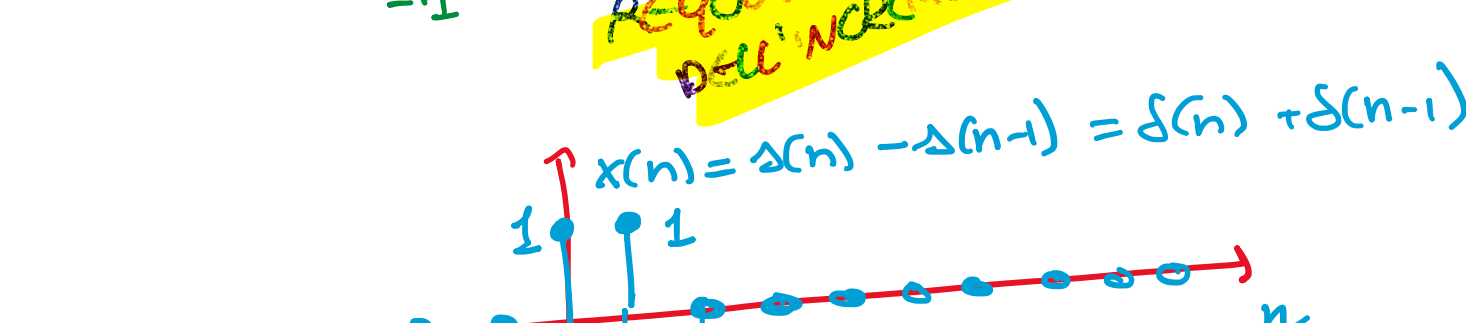
$$\theta_0' = \theta_0 + 2\pi$$

$$\cos(n\theta_0' + \theta_1) = \cos(\theta_0 n + \theta_1)$$

ES5 TROVARE $S(e^{j\omega})$ PER $\Delta(n) = \text{sign}(n)$



SIMMETRIA REALE (ODD)



$$x(n) = \Delta(n) - \Delta(n-1) = \delta(n) + \delta(n-1)$$

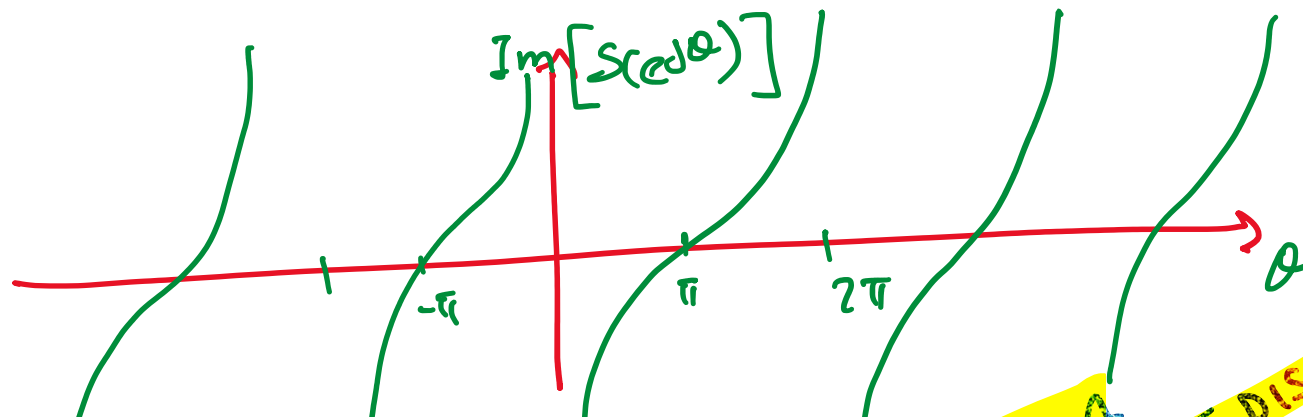
$$\Downarrow$$

$$X(e^{j\omega}) = S(e^{j\omega}) (1 - e^{-j\omega}) = 1 + e^{-j\omega}$$

$$S(e^{j\omega}) = \frac{1 + e^{-j\omega}}{1 - e^{-j\omega}} + 2\pi m_s \text{comb}(\frac{\omega}{2\pi})$$

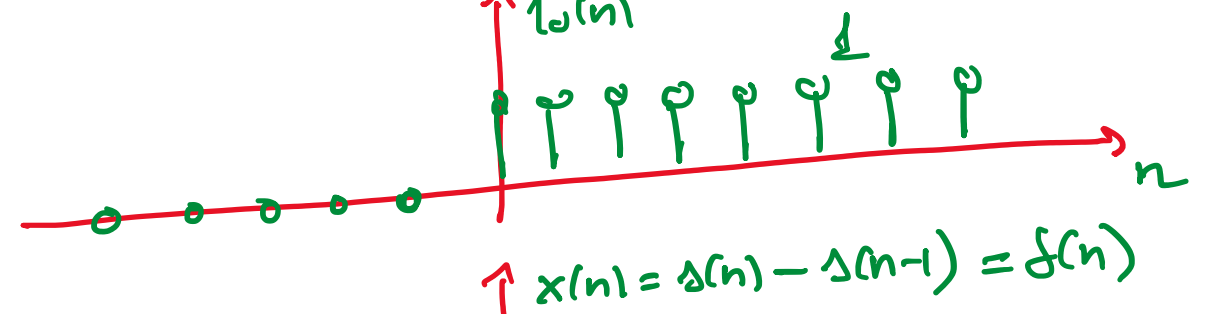
$$= \frac{1 + e^{-j\omega}}{1 - e^{-j\omega}} \frac{e^{j\omega/2}}{e^{j\omega/2}} = \frac{e^{j\omega/2} + e^{-j\omega/2}}{e^{j\omega/2} - e^{-j\omega/2}}$$

$$= \frac{2 \cos(\omega/2)}{2j \sin(\omega/2)} \frac{-j}{j} = -j \cotg(\frac{\omega}{2})$$



SIMMETRIA IMMAGINARIA E DISPARI

ES6 $\Delta(n) = \text{lo}(n)$



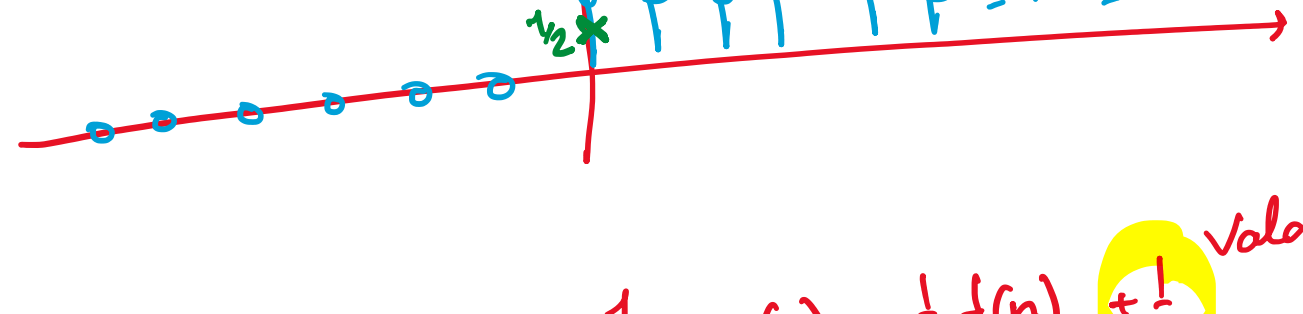
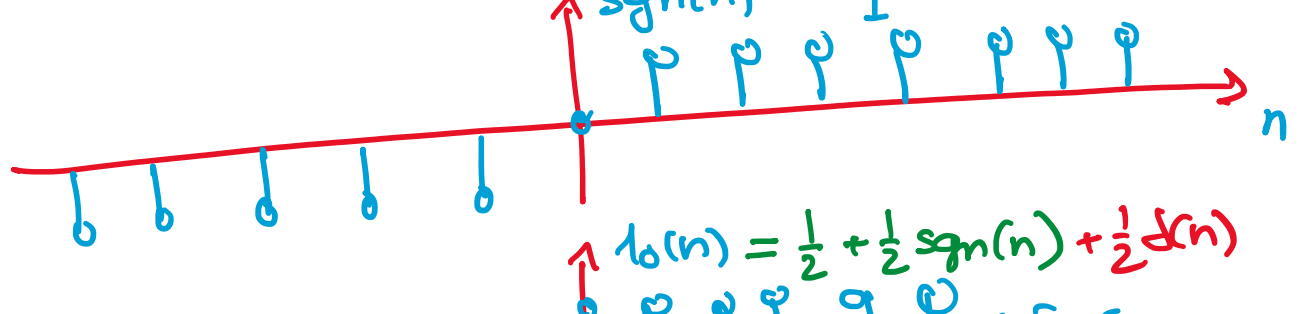
$$x(n) = \Delta(n) - \Delta(n-1) = \delta(n)$$

$$\Downarrow$$

$$S(e^{j\omega}) (1 - e^{-j\omega}) = 1$$

$$S(e^{j\omega}) = \frac{1}{1 - e^{-j\omega}} + 2\pi m_s \text{comb}(\frac{\omega}{2\pi})$$

$$m_s = \frac{1}{2}$$



$$\text{lo}(n) = \frac{1}{2} \text{sgn}(n) + \frac{1}{2} \delta(n) + \frac{1}{2}$$

$$\Downarrow$$

$$S(e^{j\omega}) = \frac{1}{2} \cdot j \cotg(\frac{\omega}{2}) + \frac{1}{2} + \frac{1}{2} \cdot 2\pi \text{comb}(\frac{\omega}{2\pi})$$

$$\frac{1}{2} \left(\frac{1 + e^{-j\omega}}{1 - e^{-j\omega}} + 1 \right) = \frac{1 + e^{-j\omega} + 1 - e^{-j\omega}}{2(1 - e^{-j\omega})} = \frac{1}{1 - e^{-j\omega}}$$