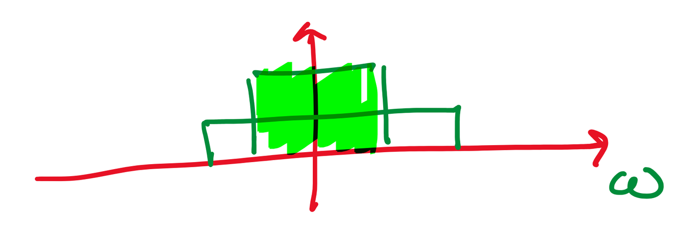
Le18

Friday, 17 April 2026 08:32

XCASA TROVARE $s(t) = x * y(t)$ CON $x(t) = \mathrm{sinc}(t/2)$

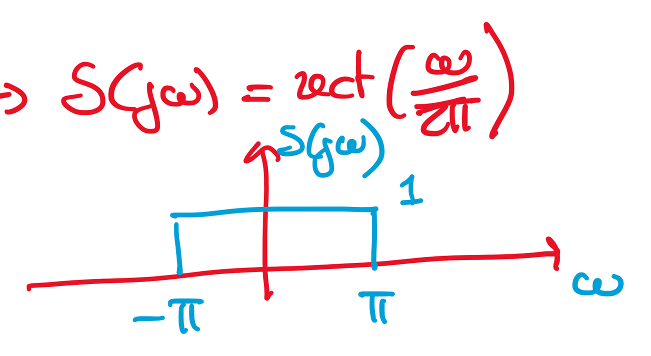
$y(t) = \mathrm{sinc}(t/3)$

SFRUTTANDO FOURIER



ES1 CALCOLARE A_s E E_s PER $s(t) = \mathrm{sinc}(t)$

$$s(t) = \mathrm{sinc}(t) \xrightarrow{\mathcal{F}} S(j\omega) = \mathrm{rect}\left(\frac{\omega}{2\pi}\right)$$



$$A_{sinc} = S(j0) = 1$$

$$E_{sinc} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |S(j\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathrm{rect}\left(\frac{\omega}{2\pi}\right)^{\cancel{2}} d\omega$$

TEOREMA DI PARSEVAL

$$= \frac{1}{2\pi}\cdot 2\pi = 1$$

ES2 CALCOLARE L'AREA DI $s(t) = \mathrm{sinc}^3(t)$

REGOLA PRODOTTO

$s(t) = x(t)\,y(t)$ $\Downarrow \mathcal{F}$

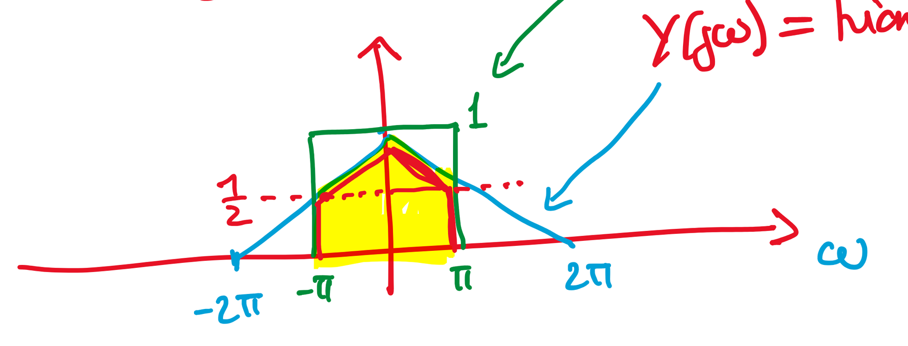
$x(t) = \mathrm{sinc}(t)$

$y(t) = \mathrm{sinc}^2(t)$

$$S(j\omega) = \frac{1}{2\pi} X * Y(j\omega)$$

$X(j\omega) = \mathrm{rect}\left(\frac{\omega}{2\pi}\right)$

$Y(j\omega) = \mathrm{triang}\left(\frac{\omega}{2\pi}\right)$



$$A_s = S(j0) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\vartheta)\, \underbrace{Y(j(\overset{0}{\cancel{\omega}}-\vartheta))}_{} d\vartheta$$

$\mathrm{rect}\left(\frac{\vartheta}{2\pi}\right)$

$Y(-j\vartheta) = \mathrm{triang}\left(\frac{-\vartheta}{2\pi}\right)$

$= \mathrm{triang}\left(\frac{\vartheta}{2\pi}\right)$

$$A_s = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\,Y(j\omega)\,d\omega = \frac{1}{\cancel{2\pi}}\cdot\frac{3}{4}\cdot\cancel{2\pi} = \frac{3}{4}$$

NOTA

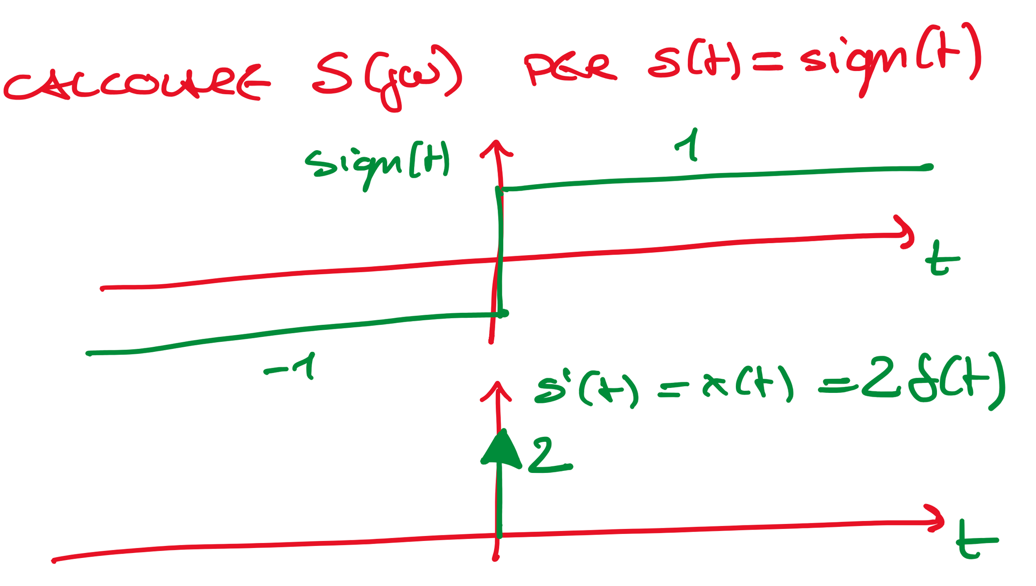
$$j\frac{\partial}{\partial\omega}S(j\omega) = \frac{\partial}{\partial\omega}\int_{-\infty}^{+\infty} s(t)\,e^{-j\omega t}\,dt \cdot j$$

$$= \cancel{j}\int_{-\infty}^{+\infty} s(t)\cdot -\cancel{j}t\,e^{-j\omega t}\,dt$$

$$j\,S'(j\omega) = \int_{-\infty}^{+\infty} s(t)\cdot t \cdot e^{-j\omega t}\,dt$$

$$\boxed{\begin{aligned} s(t) &\rightarrow S(j\omega) \\ t\cdot s(t) &\rightarrow j\,S'(j\omega) \\ t^2\cdot s(t) &\rightarrow j^2\,S''(j\omega) \end{aligned}}$$

ES3 CALCOLARE $S(j\omega)$ PER $s(t) = \mathrm{sign}(t)$



REGOLA DI DERIVAZIONE

$$x(t) = s'(t) = 2\delta(t)$$

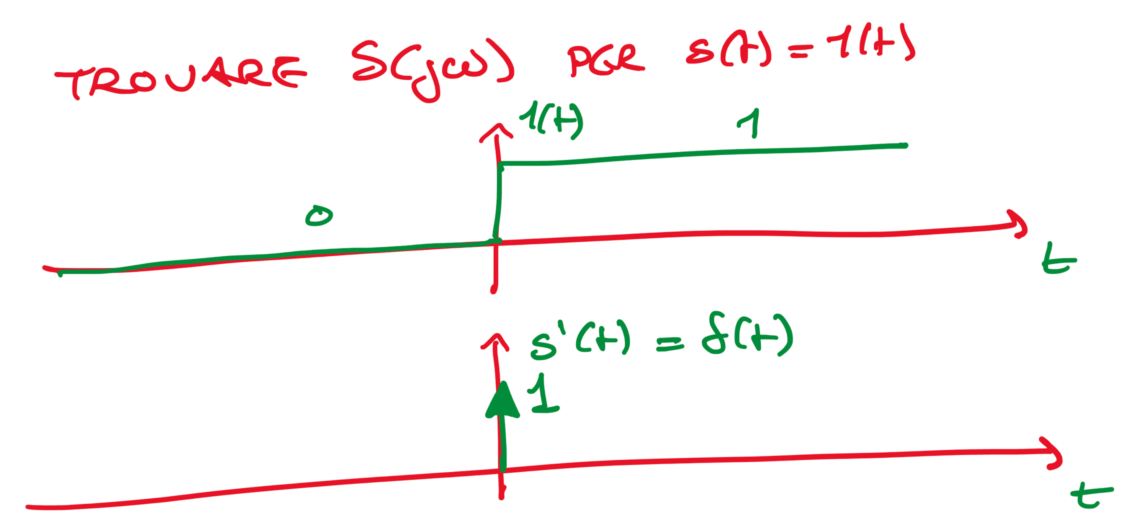
$$\mathcal{F}\downarrow \quad \downarrow\mathcal{F} \quad \downarrow\mathcal{F}$$

$$X(j\omega) = j\omega\cdot S(j\omega) = 2$$

$$S(j\omega) = \frac{2}{j\omega} + \cancel{m_s\cdot 2\pi\delta(\omega)}$$

XCHÈ $m_s = 0$

ES 4 TROVARE $S(j\omega)$ PER $s(t) = 1(t)$



$$x(t) = s'(t) = \delta(t)$$

$$\mathcal{F}\downarrow$$

$$X(j\omega) = j\omega\cdot S(j\omega) = 1$$

$$S(j\omega) = \frac{1}{j\omega} + \cancel{m_s}\cdot\cancel{2}\pi\,\delta(\omega) \qquad m_s = 1/2$$

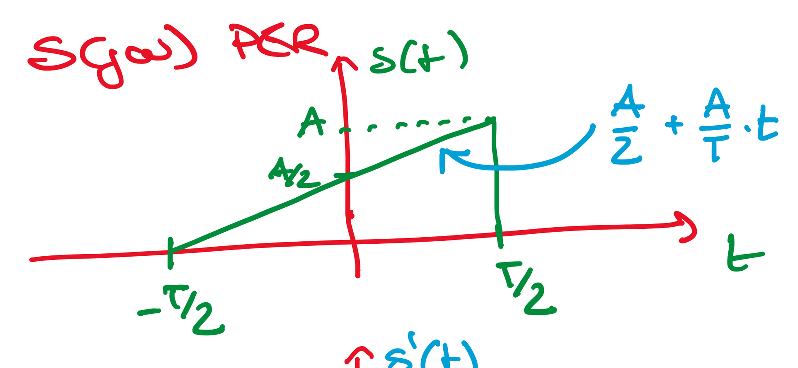
SI POTEVA ANCHE TROVARE DA

$$s(t) = 1(t) = \frac{1}{2} + \frac{1}{2}\,\mathrm{sign}(t)$$

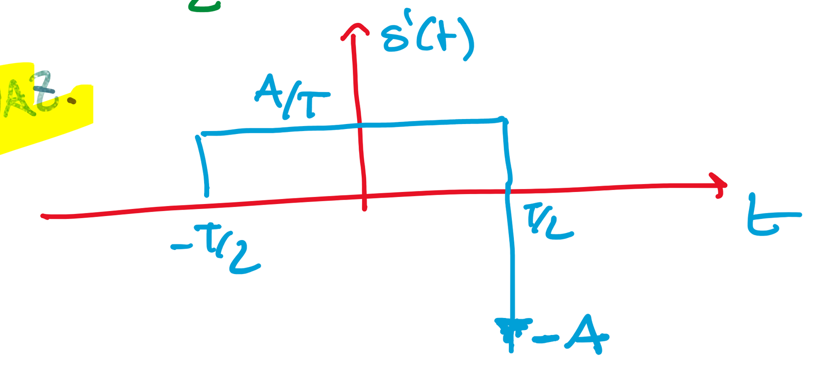
$$\mathcal{F}\downarrow$$

$$S(j\omega) = \frac{1}{\cancel{2}}\cdot\cancel{2}\pi\delta(\omega) + \frac{1}{\cancel{2}}\,\frac{\cancel{2}}{j\omega}$$

ES 5 TROVARE $S(j\omega)$ PER



REGOLA DI DERIVAZ.



$$x(t) = s'(t) = \frac{A}{T}\mathrm{rect}\left(\frac{t}{T}\right) - A\,\delta(t - T/2)$$

$$\mathcal{F}\downarrow$$

$$X(j\omega) = j\omega S(j\omega) = \frac{A}{\cancel{T}}\cdot\cancel{T}\,\mathrm{sinc}\left(\frac{\omega T}{2\pi}\right) - A\,e^{-j\omega T/2}$$

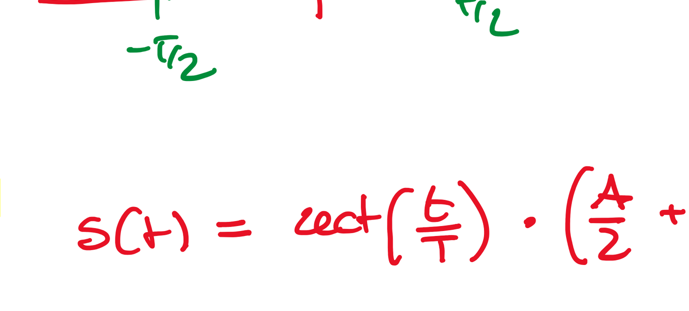
$$= A\,\mathrm{sinc}\left(\frac{\omega T}{2\pi}\right) - A\,e^{-j\omega T/2}$$

SIMMETRIA HERMITIANA

$$S(j\omega) = \frac{A\,\mathrm{sinc}(\omega T/2\pi) - A\,e^{-j\omega T/2}}{j\omega}$$

PARTE REALE

$\frac{A\sin(\omega T/2)}{\omega}$



REGOLA DERIVAZ. IN ω

$$s(t) = \mathrm{rect}\left(\frac{t}{T}\right)\cdot\left(\frac{A}{2} + \frac{A}{T}\cdot t\right)$$

$$= \underbrace{\frac{A}{2}\mathrm{rect}\left(\frac{t}{T}\right)}_{x(t)} + t\cdot\underbrace{\frac{A}{T}\mathrm{rect}\left(\frac{t}{T}\right)}_{y(t)}$$

$$S(j\omega) = X(j\omega) + j\,Y'(j\omega)$$

$$X(j\omega) = \frac{AT}{2}\,\mathrm{sinc}\left(\frac{\omega T}{2\pi}\right)$$

$$Y(j\omega) = \frac{A}{\cancel{T}}\cdot\cancel{T}\,\mathrm{sinc}\left(\frac{\omega T}{2\pi}\right)$$

$$\boxed{S(j\omega) = \frac{AT}{2}\,\mathrm{sinc}\left(\frac{\omega T}{2\pi}\right) + j\,\frac{AT}{2\pi}\,\mathrm{sinc}'\left(\frac{\omega T}{2\pi}\right)}$$

PARTE REALE

$$\frac{A\cancel{T}}{\cancel{2}}\,\frac{\sin\frac{\omega T}{2}}{\omega\frac{\cancel{T}}{\cancel{2}}}$$