Le17

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ES 1 TROVARE $S(j\omega)$ PER $s(t) = \delta(t-t_0)$

$$S(j\omega) = \int_{-\infty}^{+\infty} \delta(t-t_0)\, e^{-j\omega t}\, dt = e^{-j\omega t_0}$$

PROPRIETÀ DI TRASLAZIONE

$x(t) = \delta(t) \xrightarrow{\mathcal{F}} X(j\omega) = 1$

$x(t-t_0) = \delta(t-t_0) \xrightarrow{\mathcal{F}} X(j\omega)\, e^{-j\omega t_0} = 1 \cdot e^{-j\omega_0 t}$

ES 2 TROVARE $S(j\omega)$ PER $s(t) = e^{j\omega_1 t}$

$$S(j\omega) = \int_{-\infty}^{+\infty} e^{j\omega_1 t}\, e^{-j\omega t}\, dt = \int_{-\infty}^{+\infty} e^{-j(\omega-\omega_1)t}\, dt = ?$$

NOTA $x(t) = 1 \xrightarrow{\mathcal{F}} X(j\omega) = 2\pi\delta(\omega)$

$\mathcal{F}^{-1}$: $x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} 2\pi\delta(\omega)\, d\omega = 1$ ✓

INTUIZIONE

$X(j\omega) = 2\pi\delta(\omega-\omega_1)$

$\downarrow \mathcal{F}^{-1}$

$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} 2\pi\delta(\omega-\omega_1)\, e^{j\omega t}\, d\omega = e^{j\omega_1 t}$ ✓

REGOLA DI MODULAZIONE

$x(t) = 1 \xrightarrow{\mathcal{F}} X(j\omega) = 2\pi\delta(\omega)$

$x(t)\, e^{j\omega_1 t} = e^{j\omega_1 t} \xrightarrow{\mathcal{F}} X(j(\omega-\omega_1)) = 2\pi\delta(\omega-\omega_1)$

$\delta(t-t_0) \xrightarrow{\mathcal{F}} e^{-j\omega t_0}$

$e^{j\omega_1 t} \xrightarrow{\mathcal{F}} 2\pi\delta(\omega-\omega_1)$

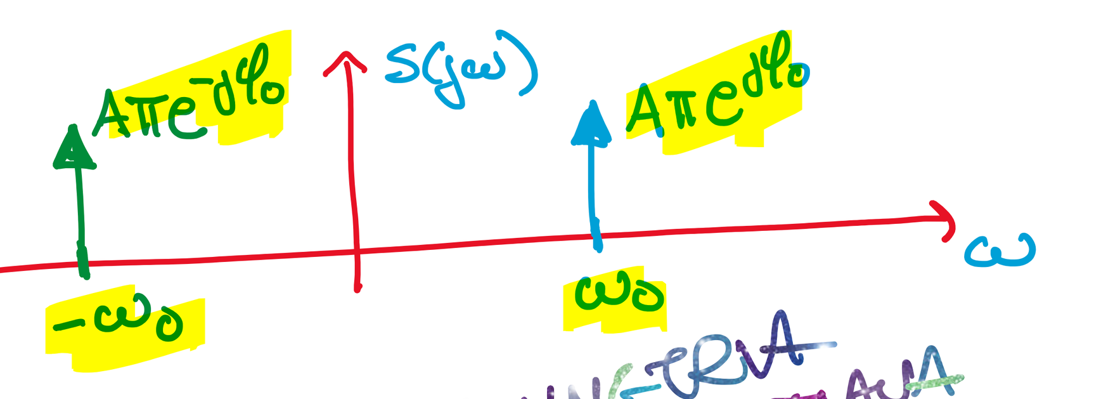
ES 3 TROVARE $S(j\omega)$ PER $s(t) = A\cos(\omega_0 t + \varphi_0)$ SEGNALE REALE

✓EULERO

$s(t) = \frac{Ae^{j\varphi_0}}{2}\cdot e^{j\omega_0 t} + \frac{Ae^{-j\varphi_0}}{2}\cdot e^{-j\omega_0 t}$

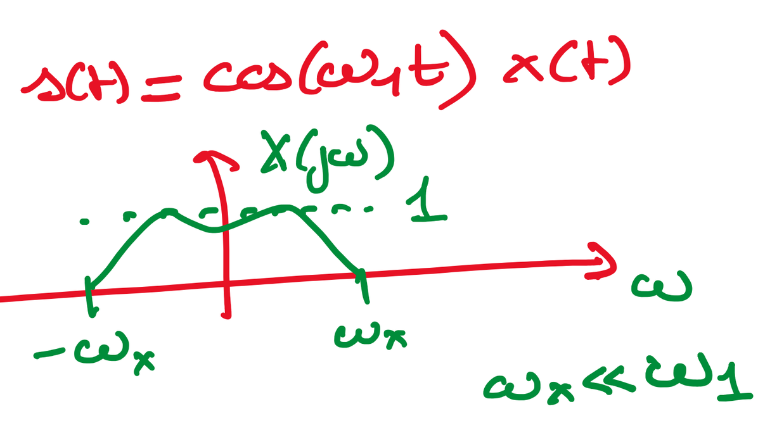
$\mathcal{F}\downarrow$

$S(j\omega) = \frac{Ae^{j\varphi_0}}{2}\, 2\pi\delta(\omega-\omega_0) + \frac{Ae^{-j\varphi_0}}{2}\, 2\pi\delta(\omega+\omega_0)$



SIMMETRIA HERMITIANA $S(-j\omega) = S^*(j\omega)$

ES 4 TROVARE $S(j\omega)$ PER $s(t) = \cos(\omega_1 t)\, x(t)$

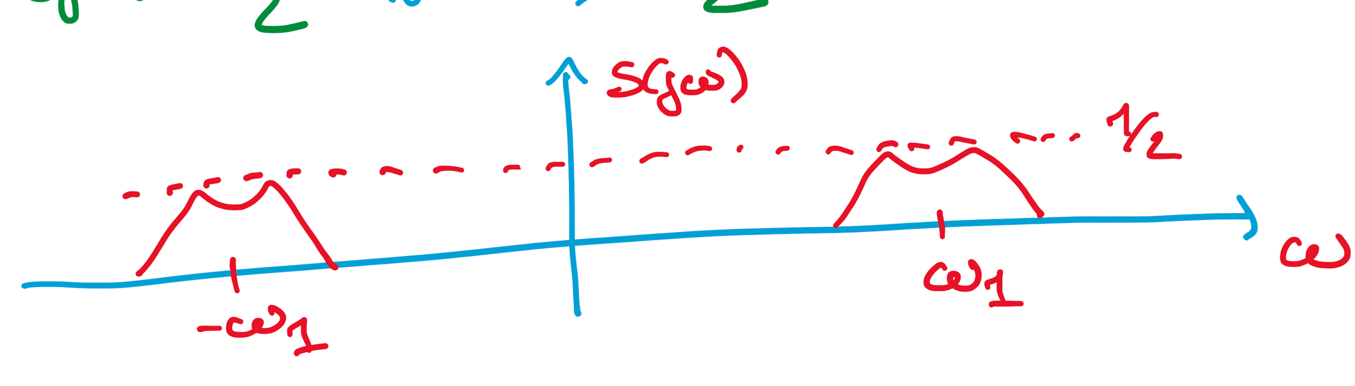


$\omega_x \ll \omega_1$

$s(t) = \frac{1}{2} x(t)\, e^{j\omega_1 t} + \frac{1}{2} x(t)\, e^{-j\omega_1 t}$

$\mathcal{F}\downarrow$

$S(j\omega) = \frac{1}{2}\cdot X(j(\omega-\omega_1)) + \frac{1}{2}\cdot X(j(\omega+\omega_1))$ REGOLA DI MODULAZIONE



REGOLA DEL PRODOTTO

$s(t) = x(t)\, y(t)$

$\mathcal{F}\downarrow$

$S(j\omega) = \frac{1}{2\pi}\, X * Y(j\omega)$

$y(t) = \cos(\omega_1 t)$

$\downarrow\mathcal{F}$

$Y(j\omega) = \pi\delta(\omega-\omega_1) + \pi\delta(\omega+\omega_1)$

$$S(j\omega) = \frac{1}{2\pi}\, X(j\omega) * [\pi\delta(\omega-\omega_1) + \pi\delta(\omega+\omega_1)]$$

$$= \frac{1}{2\pi} X(j\omega) * [\pi\delta(\omega-\omega_1)] + \frac{1}{2\pi} X(j\omega) * [\pi\delta(\omega+\omega_1)]$$

$$= \frac{1}{2} X(j(\omega-\omega_1)) + \frac{1}{2} X(j(\omega+\omega_1))$$ ✓

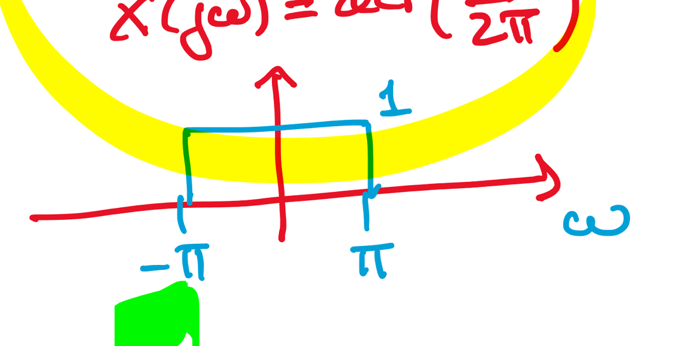
ES 5 TROVARE $S(j\omega)$ PER $s(t) = \mathrm{sinc} * \mathrm{sinc}(t)$

$s(t) = x * x(t)$ $\qquad x(t) = \mathrm{sinc}(t)$

REGOLA DI $\mathcal{F}$ CONVOLUZIONE $\downarrow$ $\qquad \downarrow\mathcal{F}$

$S(j\omega) = X(j\omega)\cdot X(j\omega)$ $\qquad X(j\omega) = \mathrm{rect}\left(\frac{\omega}{2\pi}\right)$

$= (X(j\omega))^2$



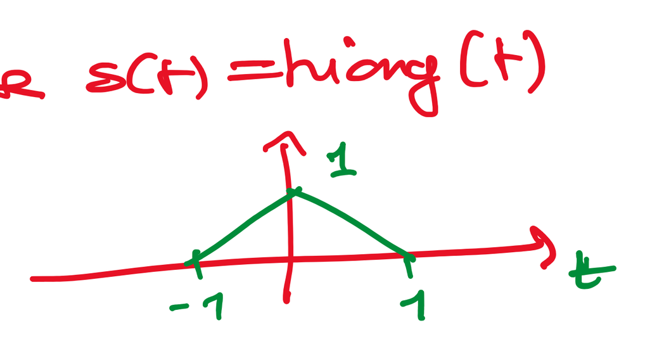
$= \left(\mathrm{rect}\left(\frac{\omega}{2\pi}\right)\right)^2$

$S(j\omega) = \mathrm{rect}\left(\frac{\omega}{2\pi}\right)$

$\mathcal{F}^{-1}\downarrow$

$s(t) = \mathrm{sinc}(t) = \mathrm{sinc} * \mathrm{sinc}(t)$

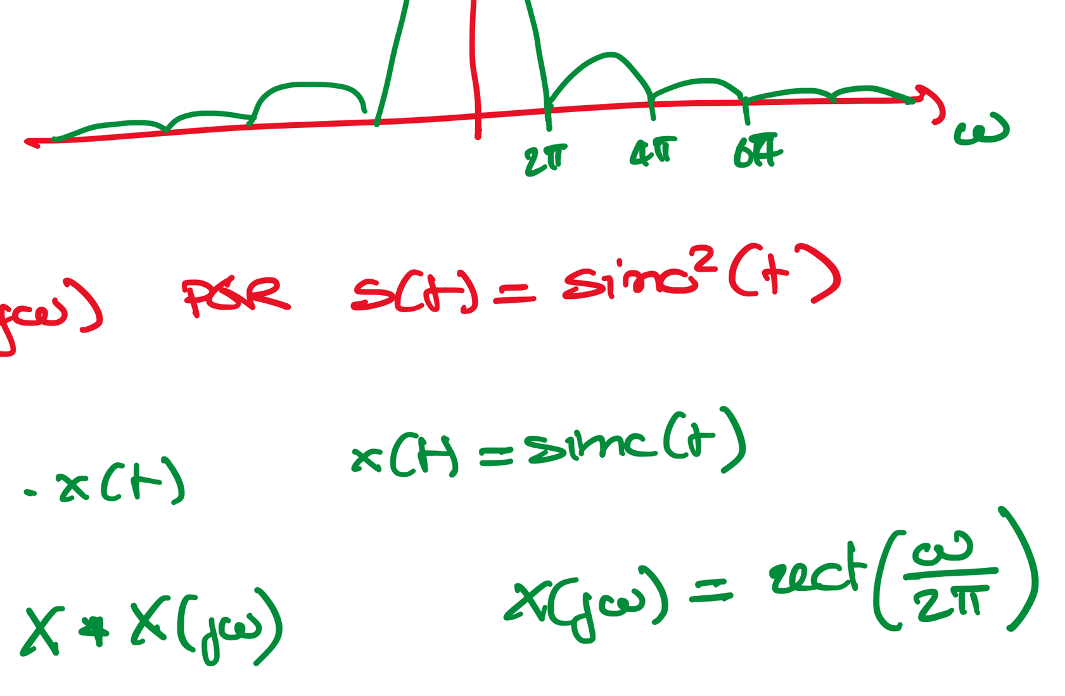
ES 6 TROVARE $S(j\omega)$ PER $s(t) = \mathrm{triang}(t)$



$s(t) = x * x(t)$ $\qquad x(t) = \mathrm{rect}(t)$

$S(j\omega) = (X(j\omega))^2$ $\qquad X(j\omega) = \mathrm{sinc}\left(\frac{\omega}{2\pi}\right)$

$= \mathrm{sinc}^2\left(\frac{\omega}{2\pi}\right)$



ES 7 TROVARE $S(j\omega)$ PER $s(t) = \mathrm{sinc}^2(t)$

$s(t) = x(t)\cdot x(t)$ $\qquad x(t) = \mathrm{sinc}(t)$

$S(j\omega) = \frac{1}{2\pi}\, X * X(j\omega)$ $\qquad X(j\omega) = \mathrm{rect}\left(\frac{\omega}{2\pi}\right)$

REGOLA DEL PRODOTTO

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathrm{rect}\left(\frac{v}{2\pi}\right) \mathrm{rect}\left(\frac{\omega - v}{2\pi}\right) dv \quad (\tfrac{v}{2\pi} \to u,\ \tfrac{dv}{2\pi} \to du)$$

$$= \int_{-\infty}^{+\infty} \mathrm{rect}(u)\, \mathrm{rect}\left(\frac{\omega}{2\pi} - u\right) du = \mathrm{triang}\left(\frac{\omega}{2\pi}\right)$$

REGOLA DI SIMMETRIA

$s(t) = \mathrm{triang}(t) \xrightarrow{\mathcal{F}} S(j\omega) = \mathrm{sinc}^2\left(\frac{\omega}{2\pi}\right)$

$x(t) = S(jt) = \mathrm{sinc}^2\left(\frac{t}{2\pi}\right)$

$\quad \hookrightarrow X(j\omega) = 2\pi\, s(-\omega)$

$\qquad = 2\pi\, \mathrm{triang}(+\omega)$

$y(t) = x\left(\frac{t}{1/2\pi}\right) = x(t\cdot 2\pi) = \mathrm{sinc}^2(t)$

$\downarrow$ REGOLA DI SCALA

$Y(j\omega) = \frac{1}{2\pi}\, X\left(j\frac{\omega}{2\pi}\right) = \frac{1}{2\pi}\cdot 2\pi\, \mathrm{triang}\left(\frac{\omega}{2\pi}\right)$

$\mathrm{rect}(t) \xrightarrow{\mathcal{F}} \mathrm{sinc}\left(\frac{\omega}{2\pi}\right)$

$\mathrm{sinc}(t) \xrightarrow{\mathcal{F}} \mathrm{rect}\left(\frac{\omega}{2\pi}\right)$

$\mathrm{triang}(t) \xrightarrow{\mathcal{F}} \mathrm{sinc}^2\left(\frac{\omega}{2\pi}\right)$

$\mathrm{sinc}^2(t) \xrightarrow{\mathcal{F}} \mathrm{triang}\left(\frac{\omega}{2\pi}\right)$

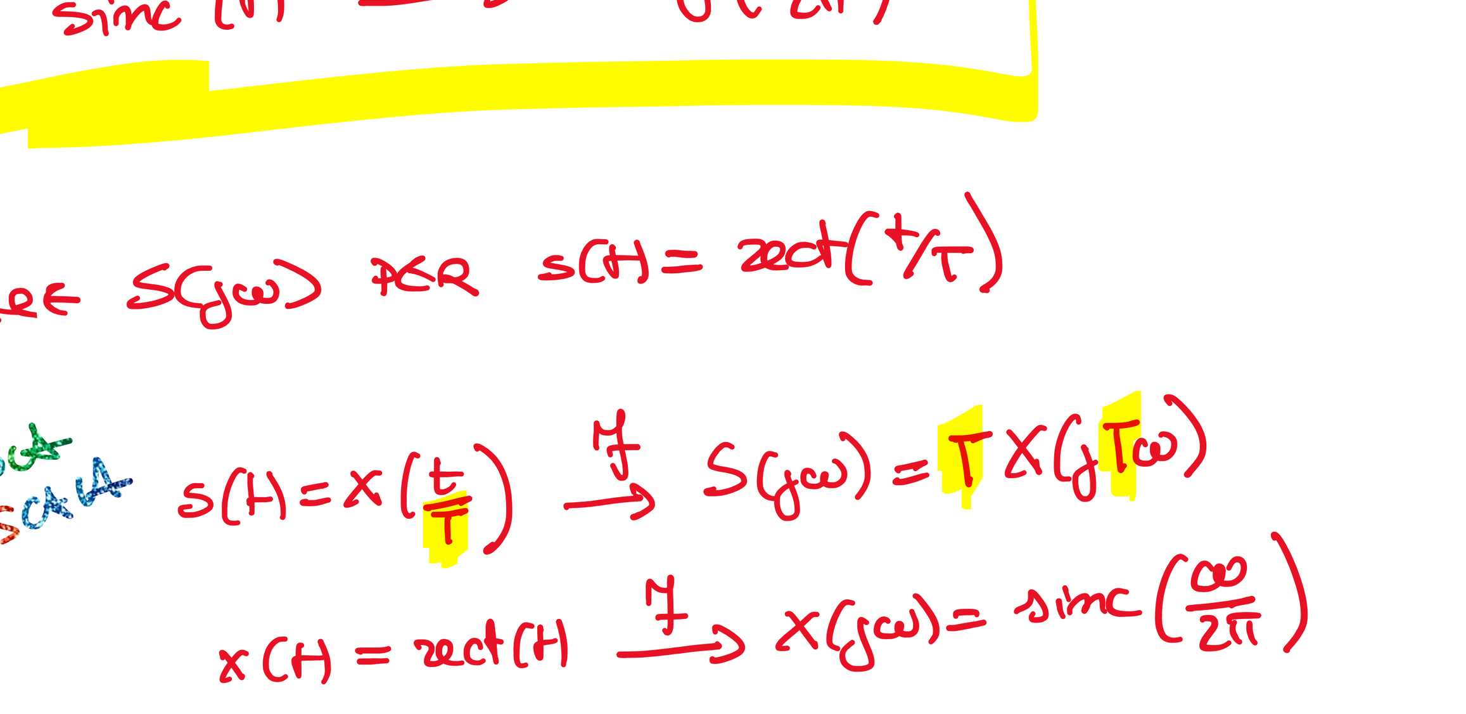
ES 8 TROVARE $S(j\omega)$ PER $s(t) = \mathrm{rect}(t/T)$

REGOLA DI SCALA $\quad s(t) = x\left(\frac{t}{T}\right) \xrightarrow{\mathcal{F}} S(j\omega) = T\, X(jT\omega)$

$x(t) = \mathrm{rect}(t) \xrightarrow{\mathcal{F}} X(j\omega) = \mathrm{sinc}\left(\frac{\omega}{2\pi}\right)$

$\mathrm{rect}\left(\frac{t}{T}\right) \xrightarrow{\mathcal{F}} T\, \mathrm{sinc}\left(\frac{T\omega}{2\pi}\right)$

$= T\, \mathrm{sinc}\left(\frac{\omega}{2\pi/T}\right)$



X CASA $\quad \mathrm{rect}\left(\frac{t-t_0}{T}\right) \xrightarrow{\mathcal{F}} ?$ $\qquad \mathrm{triang}\left(\frac{t-t_0}{T}\right) \xrightarrow{\mathcal{F}} ?$

$\mathrm{sinc}\left(\frac{t-t_0}{T}\right) \xrightarrow{\mathcal{F}} ?$ $\qquad \mathrm{sinc}^2\left(\frac{t-t_0}{T}\right) \xrightarrow{\mathcal{F}} ?$