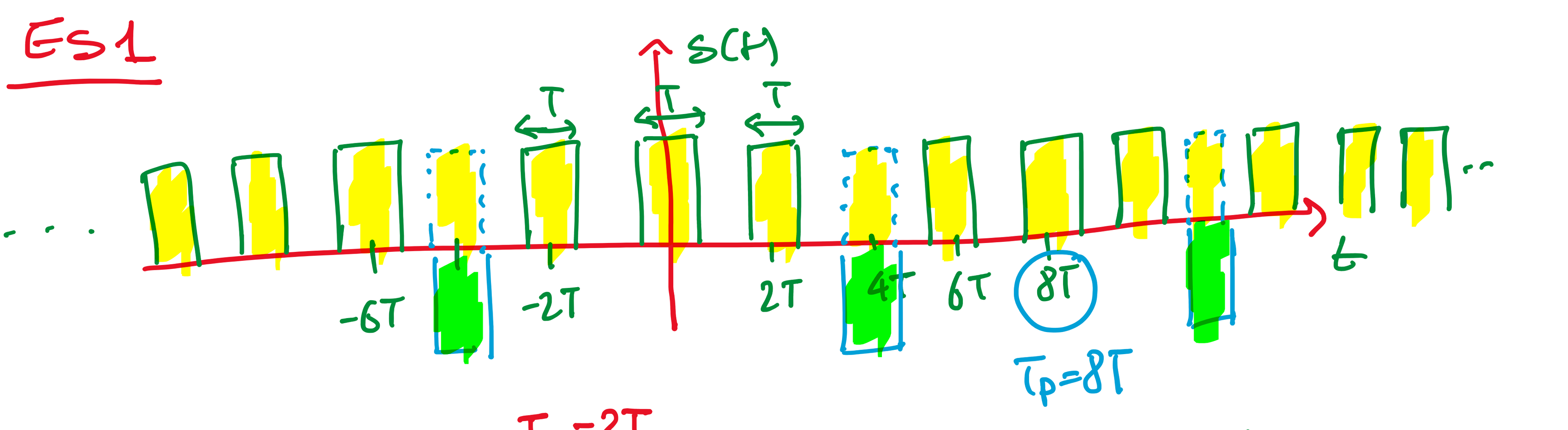
Le16

Friday, 10 April 2026 08:32

ES1



$T_p=8T$ — $T_p=2T$ ONDA QUADRA $d=\frac{1}{2}$ — ONDA QUADRA $d=\frac{1}{8}$ ~~TRASLATA~~ $t_0=4T$, $T_p=8T$

$$s(t) = rep_{2T}\, rect\left(\frac{t}{T}\right) - rep_{8T}\, rect\left(\frac{t-4T}{T}\right)$$

$s(t) \to S_K$

$rep_{2T}\, rect\left(\frac{t}{T}\right) \to X_m = \frac{1}{2}\, sinc\left(\frac{m}{2}\right)$

$rep_{8T}\, rect\left(\frac{t-4T}{T}\right) \to Y_K = \frac{1}{8}\, sinc\left(\frac{K}{8}\right) e^{-j\omega_0 K t_0}$, where $e^{-j\omega_0 K t_0} = (e^{-j\pi})^K = (-1)^K$

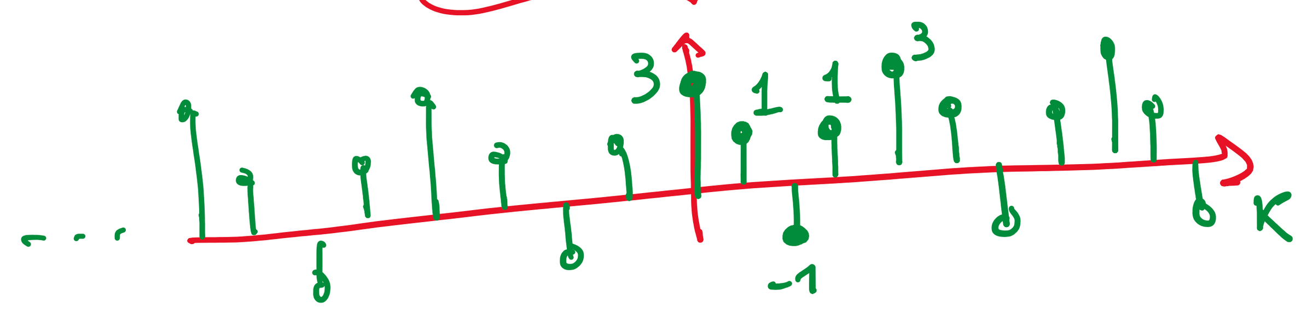
$$\omega_0 t_0 = \frac{2\pi}{8T} \cdot 4T = \pi$$

$$S_K = \begin{cases} X_m & K=4m \\ 0 & K\neq 4m \end{cases} - Y_K$$

$$= \begin{cases} \frac{1}{2}\, sinc\left(\frac{m}{2}\right) \to \frac{K}{8} & K=4m \\ 0 & K\neq 4m \end{cases} - \frac{1}{8}\, sinc\left(\frac{K}{8}\right) (-1)^K \qquad m=K/4$$

$$= \begin{cases} \frac{1}{2}\, sinc\left(\frac{K}{8}\right) - \frac{1}{8}\, sinc\left(\frac{K}{8}\right) \cancel{(-1)^K} = \frac{3}{8}\, sinc\left(\frac{K}{8}\right), & K=4m \\ -\frac{1}{8}\, sinc\left(\frac{K}{8}\right) (-1)^K & K\neq 4m \end{cases}$$

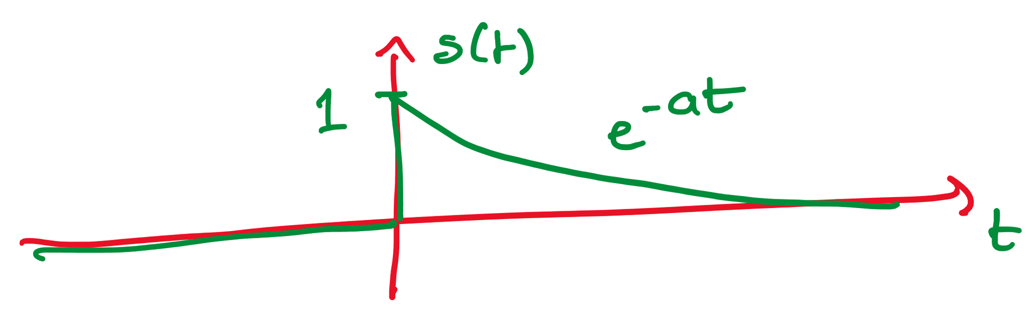
$$= \frac{1}{8}\, sinc\left(\frac{K}{8}\right) \cdot \begin{cases} 3 & K=4m \\ (-1)^{K+1} & K\neq 4m \end{cases}$$



$$1 + 2\cos\left(\frac{\pi}{2}K\right)$$

ES2

TROVARE $S(j\omega)$ PER $s(t) = e^{-at}\,1(t)$, $a>0$

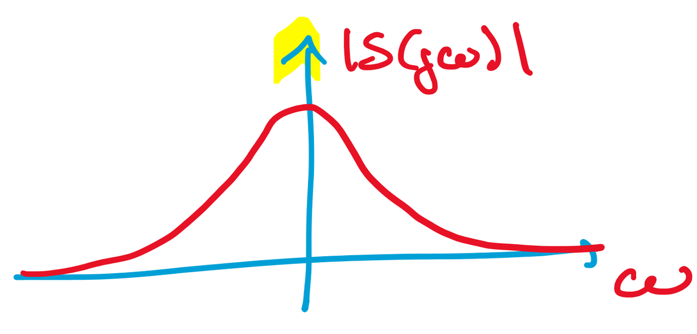


$$S(j\omega) = \int_{\cancel{-\infty}\;0}^{+\infty} \cancel{s(t)}\, e^{-at}\, e^{-j\omega t}\, dt$$

$$= \int_0^{\infty} e^{-(a+j\omega)t}\, dt = \left.\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right]_0^{\infty}$$

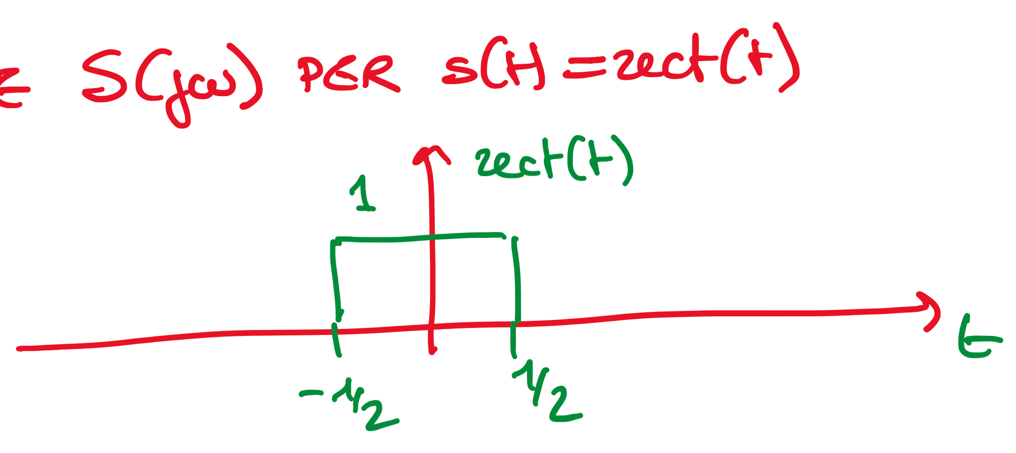
$$= \frac{0-1}{-(a+j\omega)} = \frac{1}{a+j\omega}\,\frac{(a-j\omega)}{(a-j\omega)}$$

$$S(j\omega) = \frac{a-j\omega}{a^2+\omega^2}$$



ES 3

TROVARE $S(j\omega)$ PER $s(t) = rect(t)$



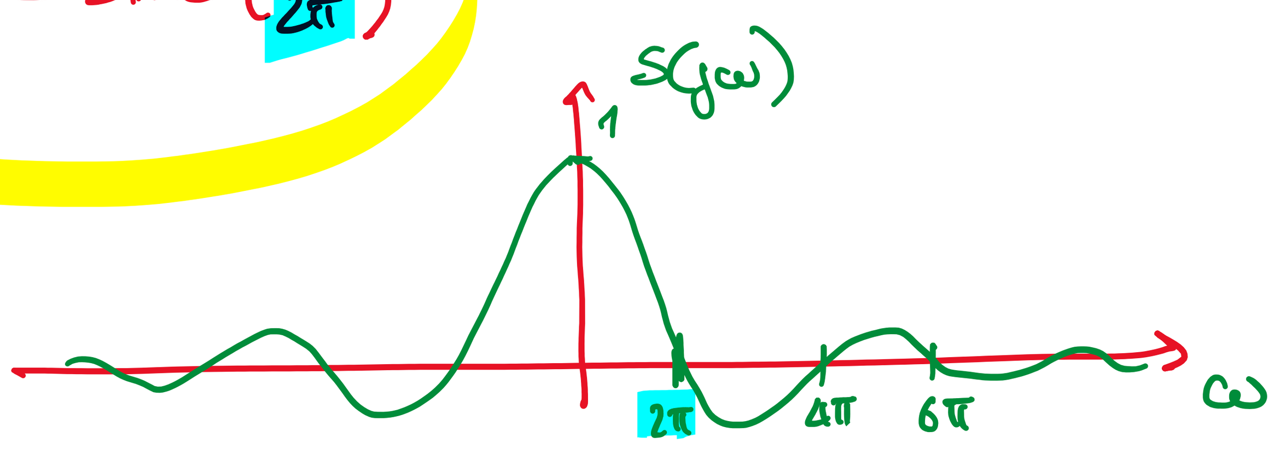
$$S(j\omega) = \int_{\cancel{-\infty}\;-\frac{1}{2}}^{\cancel{+\infty}\;\frac{1}{2}} \cancel{s(t)}\, e^{-j\omega t}\, dt = \int_{-1/2}^{1/2} e^{-j\omega t}\, dt$$

$$= \begin{cases} \int_{-1/2}^{1/2} 1\, dt = 1 & \omega = 0 \\ \left.\frac{e^{-j\omega t}}{-j\omega}\right]_{-1/2}^{1/2} & \omega \neq 0 \end{cases}$$

$$\frac{-e^{-j\omega/2} + e^{j\omega/2}}{+j\omega} = \frac{\cancel{2}\cancel{j}\sin\left(\frac{\omega}{2\pi}\cdot\pi\right)}{\cancel{j}\,\frac{\omega}{2\pi}\cdot\pi}$$

$$= sinc\left(\frac{\omega}{2\pi}\right)$$

$s(t) = rect(t)$
$\downarrow \mathcal{F}$
$S(j\omega) = sinc\left(\frac{\omega}{2\pi}\right)$



ES4

$s(t) = \delta(t)$

$$S(j\omega) = \int_{-\infty}^{+\infty} \delta(t)\, e^{-j\omega t}\, dt = \left. e^{-j\omega t}\right|_{t=0} = 1$$

$$\delta(t) \xrightarrow{\mathcal{F}} 1$$

ES 5

USARE SCALA E SIMMETRIA CON $\delta(t) \xrightarrow{\mathcal{F}} 1$

$x(t) = \delta(t) \xrightarrow{\mathcal{F}} X(j\omega) = 1$

TEMPO — FOURIER

$y(t) = X(jt) = 1 \xrightarrow{\mathcal{F}} Y(j\omega) = 2\pi\, x(-\omega)$
$= 2\pi\, \delta(-\omega)$
$= 2\pi\, \delta(\omega)$

$\delta(t) \xrightarrow{\mathcal{F}} 1$
$1 \xrightarrow{\mathcal{F}} 2\pi\,\delta(\omega)$

CONTROLLO

$$\int_{-\infty}^{+\infty} 1 \cdot e^{-j\omega t}\, dt = ?$$

PER ANTITRASFORMATA ✓

$$\frac{1}{\cancel{2\pi}} \int_{-\infty}^{+\infty} \cancel{2\pi}\,\delta(\omega)\, e^{j\omega t}\, d\omega = 1$$