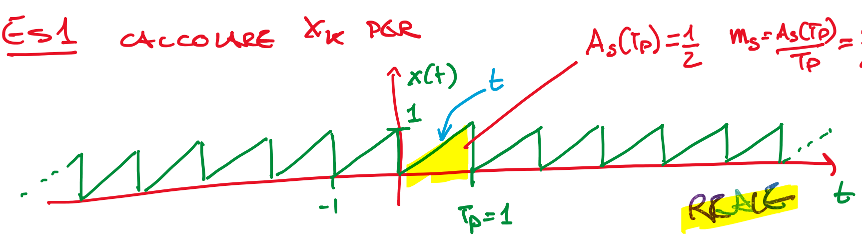
Le15

Thursday, 9 April 2026 08:30

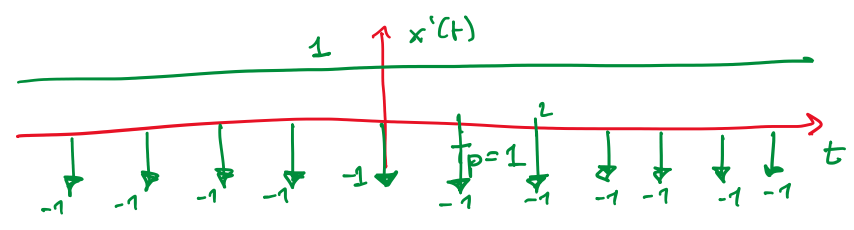
ES1 CALCOLARE X_K PER

$A_s(T_p)=\frac{1}{2}$ $m_s=\frac{A_s(T_p)}{T_p}=\frac{1}{2}$



$T_p=1 \quad \omega_0=\frac{2\pi}{T_p}=2\pi$

$$X_K=\int_0^1 t\, e^{-jK\omega_0 t}\,dt=\dots$$



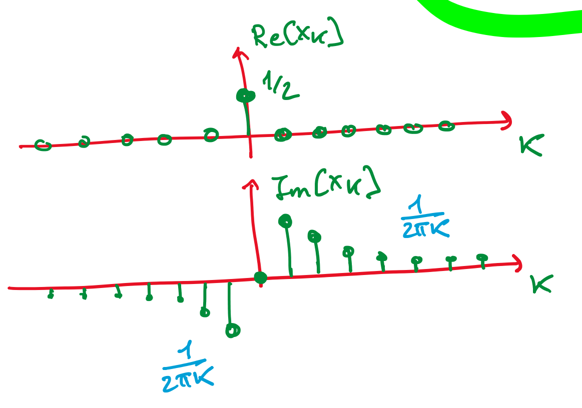
$$y(t)=x'(t)=1-rep_1\delta(t)=1-comb_1(t)$$

$$Y_K=\delta(K)-1=\begin{cases}0 & K=0\\ -1 & K\neq 0\end{cases}$$

$$=X_K\cdot j\omega_0 K=X_K\cdot j2\pi K$$

$$X_K=\begin{cases}\frac{Y_K}{j2\pi K}=\frac{-1}{j2\pi K}=\frac{j}{2\pi K} & K\neq 0\\ m_s=\frac{1}{2} & K=0\end{cases}$$

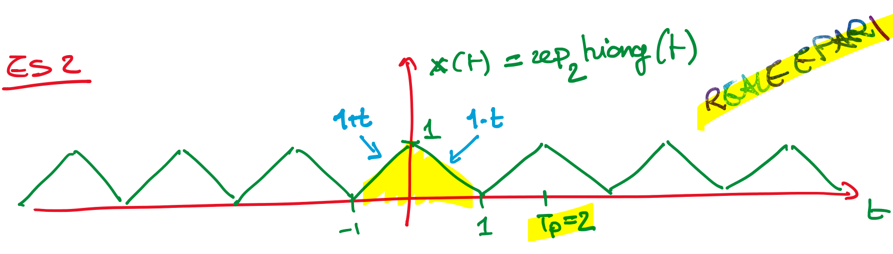
SIMM. HERMITIANA
PARTE REALE PARI
PARTE IMMAGINARIA DISPARI



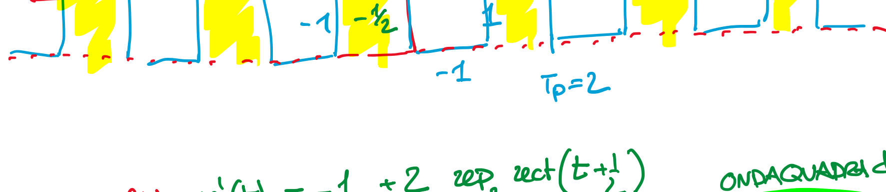
ES2

$x(t)=rep_2\,triang(t)$

REALE E PARI



$\omega_0=\frac{2\pi}{T_p}=\pi$



$$y(t)=x'(t)=-1+2\,rep_2\,rect\left(t+\frac{1}{2}\right)$$

ONDAQUADRA $d=\frac{1}{2}$

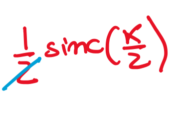
$$=-1+2\,u(t+1/2)$$

$u(t)=rep_2\,rect(t)$

$$Y_K=-1\cdot\delta(K)+2\cdot U_K\cdot e^{-jK\omega_0\cdot(-\frac{1}{2})}$$

$$=-\delta(K)+2\cdot\frac{1}{2}sinc\left(\frac{K}{2}\right)\cdot e^{jK\frac{\pi}{2}}$$

$(e^{j\pi/2})^K$, j



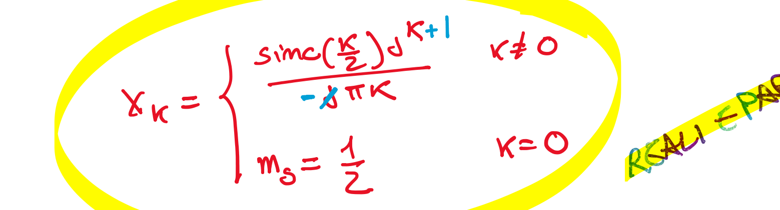
$$Y_K=\begin{cases}-1+1=0 & K=0\\ sinc\left(\frac{K}{2}\right)\cdot j^K & K\neq 0\end{cases}$$

$$=X_K\cdot jK\omega_0=X_K\cdot j\pi K$$

$$X_K=\begin{cases}\frac{sinc(\frac{K}{2})j^{K+1}}{-\pi K} & K\neq 0\\ m_s=\frac{1}{2} & K=0\end{cases}$$

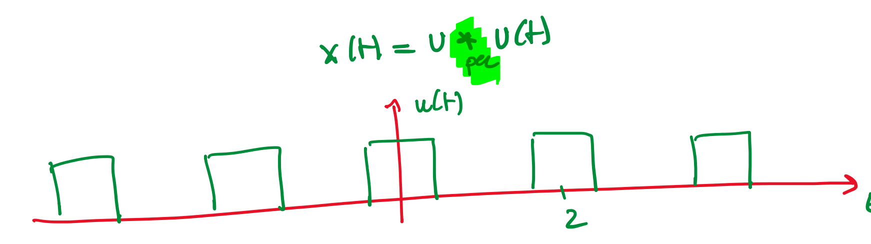
REALI E PARI

$sinc(K/2)\,j^{K+1}$ $K\neq 0$



SOLUZIONE ALTERNATIVA

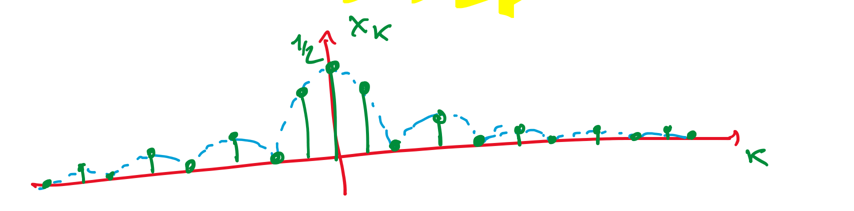
$x(t)=rep_2\,triang(t)$



$triang(t)=rect*rect(t)$

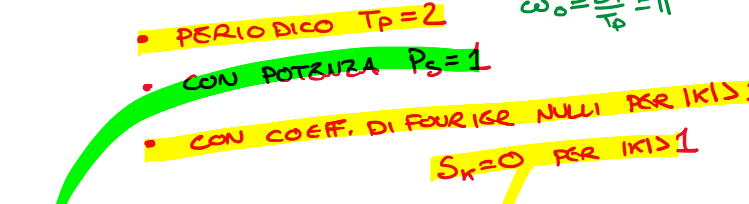
$u(t)=rep_2\,rect(t)=$ ONDAQUADRA $d=\frac{1}{2}$

$x(t)=u \underset{per}{*} u(t)$



$$X_K=T_p\cdot U_K\cdot U_K=2\left(\frac{1}{2}sinc\left(\frac{K}{2}\right)\right)^2$$

$$=\frac{1}{2}sinc^2\left(\frac{K}{2}\right)$$



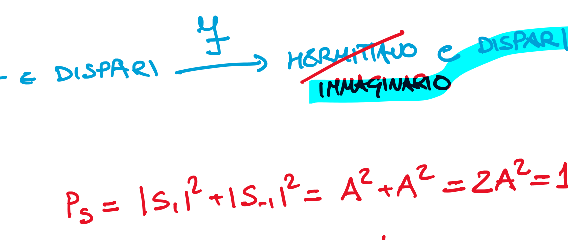
ES3 SIA DATO s(t) TALE CHE

- REALE E DISPARI → $m_s=0 \rightarrow S_0=0$
- PERIODICO $T_p=2$ $\omega_0=\frac{2\pi}{T_p}=\pi$
- CON POTENZA $P_s=1$
- CON COEFF. DI FOURIER NULLI PER $|K|>1$

$S_K=0$ PER $|K|>1$

IDENTIFICARE s(t).

$S_1=jA$
$S_{-1}=-jA$



$$s(t)=\cancel{S_0}+S_1e^{j\pi t}+S_{-1}e^{-j\pi t}$$

$$P_s=\cancel{|S_0|^2}+|S_1|^2+|S_{-1}|^2=1$$

REALE E DISPARI $\xrightarrow{\mathcal{F}}$ ~~HERMITIANO~~ IMMAGINARIO E DISPARI

$$P_s=|S_1|^2+|S_{-1}|^2=A^2+A^2=2A^2=1$$

$$A^2=\frac{1}{2} \qquad A=\pm\frac{1}{\sqrt{2}}$$

$$s(t)=jAe^{j\pi t}-jAe^{-j\pi t}$$

$$=jA\frac{\left(e^{j\pi t}-e^{-j\pi t}\right)}{2j}\cdot 2j$$

$$=2Aj^2\sin(\pi t)$$

$$=-2A\sin(\pi t)$$

$$s(t)=\pm\sqrt{2}\sin(\pi t)$$

ES4 SIA DATO IL SEGNALE $s(t)=\frac{3}{5}\frac{\sin(\pi t)}{\sin(\frac{\pi}{5}t)}$ SINC PERIODICO

TROVARE
1) PERIODO T_p
2) S_K
3) P_s, m_s

$$s(t)=\frac{3}{5}\frac{e^{j\pi t}-e^{-j\pi t}}{\cancel{2j}}\cdot\frac{\cancel{2j}}{e^{j\frac{\pi}{5}t}-e^{-j\frac{\pi}{5}t}}$$

$$=\frac{3}{5}\frac{\cancel{e^{j\pi t}}\left(1-e^{-j2\pi t}\right)}{\cancel{e^{j\frac{\pi}{5}t}}\left(1-e^{-j\frac{2\pi}{5}t}\right)} \quad e^{j\frac{4}{5}\pi t}$$

$\alpha=e^{-j\frac{2\pi}{5}t}$

$$=\frac{3}{5}\cdot\frac{1-\alpha^5}{1-\alpha}\cdot\alpha^{-2}$$

$$=\frac{3}{5}\left(1+\alpha+\alpha^2+\alpha^3+\alpha^4\right)\cdot\alpha^{-2}$$

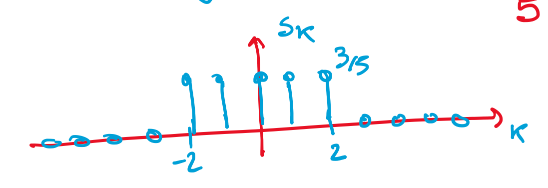
$$=\frac{3}{5}\left(\alpha^{-2}+\alpha^{-1}+1+\alpha+\alpha^2\right)$$

$$=\frac{3}{5}\sum_{K=-2}^{2}\alpha^K=\frac{3}{5}\sum_{K=-2}^{2}e^{-j\frac{2\pi}{5}Kt}$$

$-K\rightarrow m$

$$s(t)=\frac{3}{5}\sum_{m=-2}^{2}e^{j\frac{2\pi}{5}mt}$$

$\omega_0=\frac{2\pi}{5}\rightarrow T_p=5$



$m_s=3/5$

$$P_s=5\cdot\left(\frac{3}{5}\right)^2=\frac{9}{5}$$