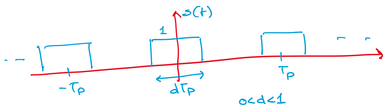
Le14

Monday, 30 March 2026 08:32

ES 1

CALCOLARE P_S CON S ONDA QUADRA NEL DOMINIO DI FOURIER



$0<d<1$

$$s(t) = rep_{T_p}\, rect\left(\frac{t}{dT_p}\right)$$

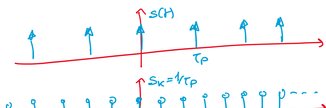
$$S_k = d\, sinc(kd)$$

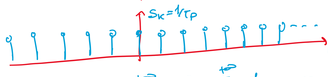
$$P_S = E_{S_k} = \sum_{k=-\infty}^{+\infty} |S_k|^2 = \sum_{k=-\infty}^{+\infty} d^2\, sinc^2(kd)$$

$$= m_{|S|^2} = m_S = \frac{d \cdot T_p}{T_p} = d$$

ES 2

$$s(t) = comb_{T_p}(t) = rep_{T_p}\, \delta(t)$$





$$P_S = E_{S_k} = \sum_{k=-\infty}^{+\infty} |S_k|^2 = \sum_{k=-\infty}^{+\infty} \frac{1}{T_p^2} = \infty$$

NON SI PUÒ PROVARE NECESSARIO

ES 3

$s(t) = 1$ $\quad S_k = \delta(k)$

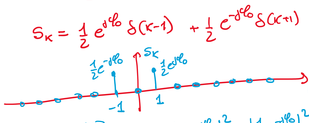
$$P_S = E_{S_k} = \sum_{k=-\infty}^{+\infty} |S_k|^2 = 1$$

ES 4

$$s(t) = \cos(\omega_0 t + \varphi_0) \qquad \omega_0 = 2\pi/T_p$$

$$= \frac{1}{2} e^{j\varphi_0} e^{j\omega_0 t} + \frac{1}{2} e^{-j\varphi_0} e^{-j\omega_0 t}$$

$$S_k = \frac{1}{2} e^{j\varphi_0}\, \delta(k-1) + \frac{1}{2} e^{-j\varphi_0}\, \delta(k+1)$$



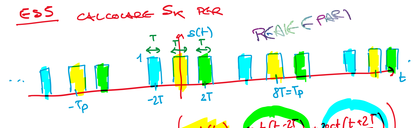
$$P_S = \sum_{k=-\infty}^{+\infty} |S_k|^2 = \left|\frac{1}{2} e^{j\varphi_0}\right|^2 + \left|\frac{1}{2} e^{-j\varphi_0}\right|^2$$

$$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

ES 5

CALCOLARE S_k PER

REALE E PARI



$$s(t) = rep_{T_p = 8T}\left(rect\left(\frac{t}{T}\right) + rect\left(\frac{t-2T}{T}\right) + rect\left(\frac{t+2T}{T}\right)\right)$$

$$u(t) = rep_{T_p = 8T}\, rect\left(\frac{t}{T}\right)$$ ONDA QUADRA $d = \frac{T}{8T} = \frac{1}{8}$

$$U_k = d\, sinc(kd) = \frac{1}{8} sinc\left(\frac{k}{8}\right)$$

$u(t-2T)$ $\quad u(t+2T)$

$$s(t) = u(t) + u(t-2T) + u(t+2T)$$

$$S_k = U_k + U_k e^{-jk\omega_0 2T} + U_k e^{-jk\omega_0(-2T)} \qquad \omega_0 = \frac{2\pi}{T_p} = \frac{\pi}{4T}$$

$$= U_k\left(1 + e^{-jk\omega_0 2T} + e^{jk\omega_0 2T}\right)$$

$$= U_k\left(1 + 2\cos(k\underbrace{\omega_0 2T}_{\pi/2})\right)$$

$$\boxed{S_k = \frac{1}{8} sinc\left(\frac{k}{8}\right)\left(1 + 2\cos\left(k\frac{\pi}{2}\right)\right)}$$

REALE E PARI

ES 6

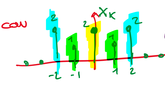
CALCOLARE S_k PER $s(t) = \cos(\omega_0 t + \varphi_0)$

$$s(t) = \underbrace{\frac{1}{2} e^{j\varphi_0}}_{\text{SEGNALE COSTANTE}} \overbrace{e^{j\omega_0 t}}^{\text{MODULAZIONE } m=1} + \overbrace{\frac{1}{2} e^{-j\varphi_0} \cdot e^{-j\omega_0 t}}^{\text{MODULAZIONE } m=-1} \qquad \omega_0 = \frac{2\pi}{T_p}$$

$$S_k = \frac{1}{2} e^{j\varphi_0}\, \delta(k-1) + \frac{1}{2} e^{-j\varphi_0}\, \delta(k+1)$$

ES 7

SIA DATO $x(t)$ CON PERIODICO T_p



REALE E PARI

1) TROVARE $x(t)$

2) CALCOLARE S_k PER $s(t) = x(t)\cos(10\omega_0 t)$

$\omega_0 = 2\pi/T_p$

$$x(t) = \sum_{k=-\infty}^{+\infty} X_k e^{jk\omega_0 t} = 2 + 1 \cdot e^{j\omega_0 t} + 1 \cdot e^{-j\omega_0 t} + 2 e^{j2\omega_0 t} + 2 e^{-j2\omega_0 t}$$

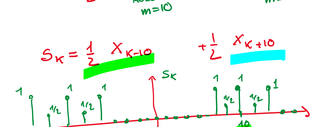
$$= 2 + 2\cos(\omega_0 t) + 4\cos(2\omega_0 t)$$

REALE E PARI

$$s(t) = x(t)\cos(10\omega_0 t)$$

$$= \frac{1}{2} x(t) \underbrace{e^{j10\omega_0 t}}_{\text{MODULAZIONE } m=10} + \frac{1}{2} x(t) \underbrace{e^{-j10\omega_0 t}}_{\text{MODULAZIONE } m=-10}$$

$$S_k = \frac{1}{2} X_{k-10} + \frac{1}{2} X_{k+10}$$



3) TROVARE LA POTENZA DI $x(t)$ E $s(t)$

$$P_x = \sum_{k=-\infty}^{+\infty} |X_k|^2 = 3 \cdot 2^2 + 2 \cdot 1^2 = 14$$

$$P_S = \sum_{k=-\infty}^{+\infty} |S_k|^2 = 6 \cdot 1^2 + 4 \cdot \frac{1}{4} = 7 = \frac{P_x}{2}$$