Le13

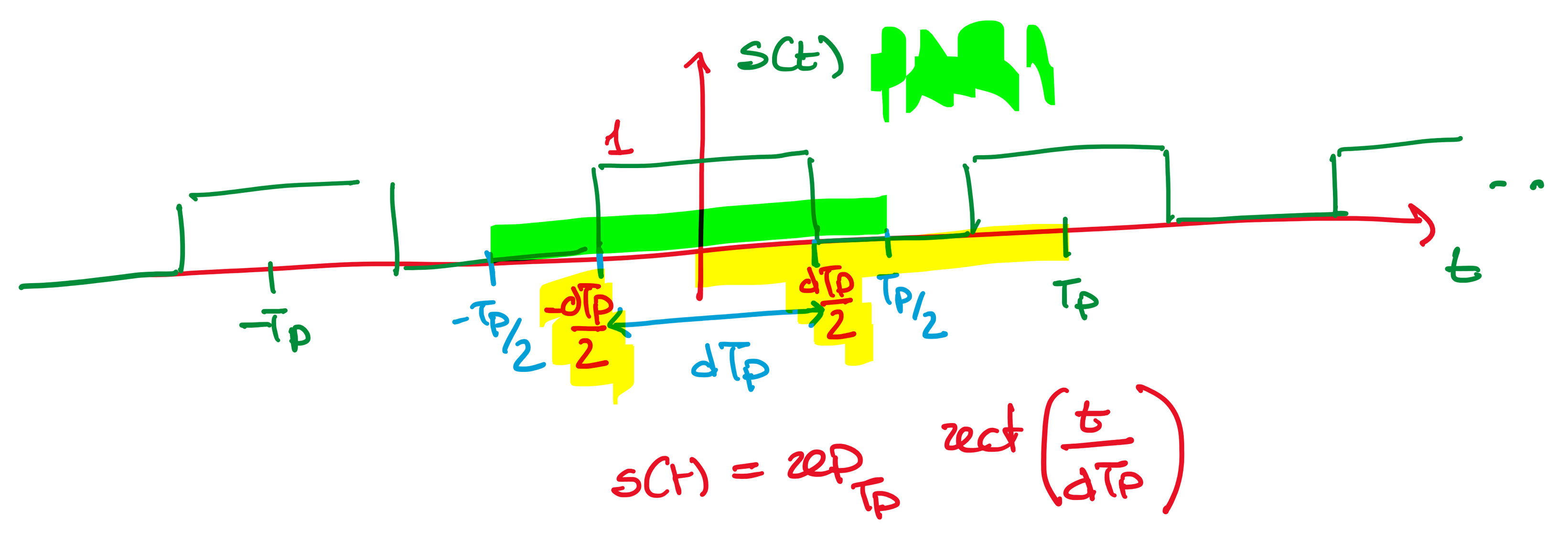
Friday, 27 March 2026 08:35

ES 1

SIA $s(t)$ ONDA QUADRA, PERIODO T_P, DUTY CYCLE d

CALCOLARE S_K $\qquad 0<d<1$

PARI



$$s(t) = rep_{T_P}\ rect\left(\frac{t}{dT_P}\right)$$

$\omega_0 = \frac{2\pi}{T_P}$

$e^{jK\omega_0 t}$ ← PERIODICI T_P

$$S_K = \frac{1}{T_P}\int_{-\frac{T_P}{2}}^{T_P/2} s(t)\, e^{-jK\omega_0 t}\, dt$$

$$= \frac{1}{T_P}\int_{-dT_P/2}^{dT_P/2} rect\left(\frac{t}{dT_P}\right) e^{-jK\omega_0 t}\, dt$$

$$= \frac{1}{T_P}\int_{-\frac{dT_P}{2}}^{\frac{dT_P}{2}} e^{-jK\omega_0 t}\, dt \qquad K\neq 0$$

$$= \frac{1}{T_P}\left[\frac{e^{-jK\omega_0 t}}{-jK\omega_0}\right]_{-\frac{dT_P}{2}}^{\frac{dT_P}{2}}$$

$$= \frac{e^{-jK d\pi} - e^{jK d\pi}}{-jK\, 2\pi}$$

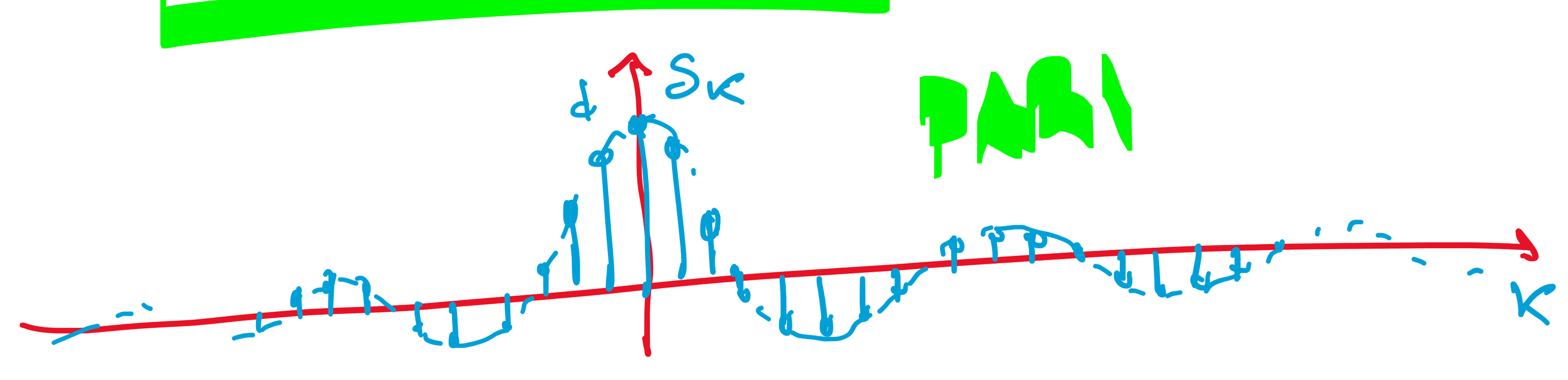
$$= \frac{-e^{-jKd\pi} + e^{jKd\pi}}{+j\, 2\pi K} \qquad \sin\alpha = \frac{e^{j\alpha}-e^{-j\alpha}}{2j}$$

$$= \frac{\sin(Kd\pi)}{\pi K d}\cdot d = d\ \mathrm{sinc}(Kd)$$

$$S_K = \begin{cases} \frac{1}{T_P}\int_{-\frac{dT_P}{2}}^{\frac{dT_P}{2}} 1\, dt = d, & K=0 \\ d\ \mathrm{sinc}(Kd), & K\neq 0 \end{cases}$$

$$S_K = d\ \mathrm{sinc}(Kd)$$

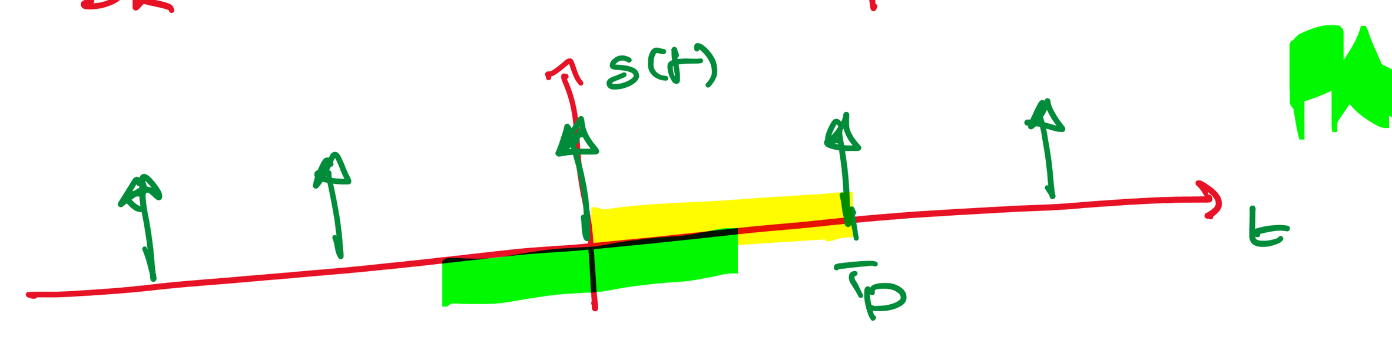
PARI



ES 2

S_K PER $s(t) = comb_{T_P}(t) = rep_{T_P}\ \delta(t)$

PARI



$$S_K = \frac{1}{T_P}\int_{-T_P/2}^{T_P/2} \delta(t)\, e^{-jK\omega_0 t}\, dt \qquad \omega_0 = \frac{2\pi}{T_P}$$

$$= \frac{e^{-jK\omega_0\cdot 0}}{T_P} = \frac{1}{T_P}$$

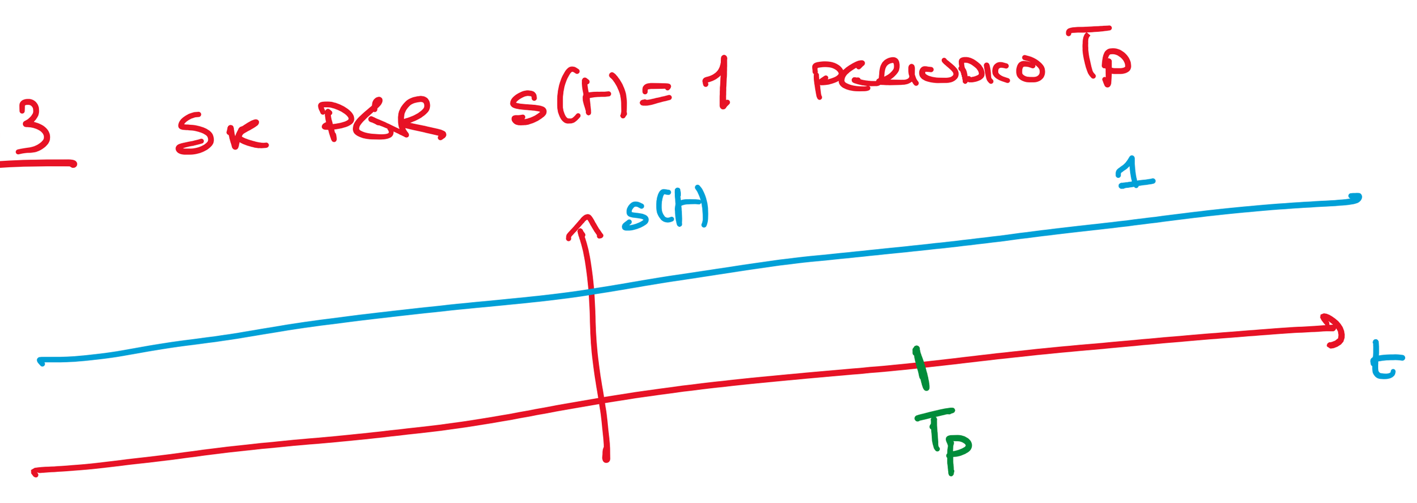
PARI

$$s(t) = rep_{T_P}\ \delta(t) = \sum_{K=-\infty}^{+\infty} \frac{1}{T_P} e^{jK\omega_0 t} \qquad \omega_0 = \frac{2\pi}{T_P}$$

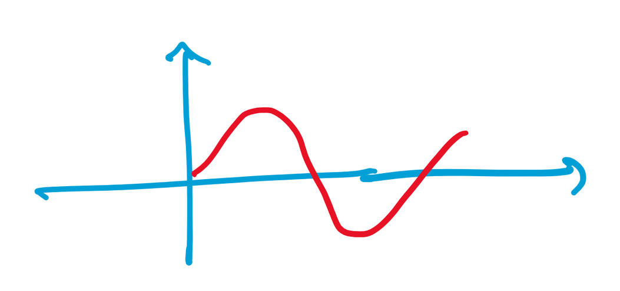
ES 3

S_K PER $s(t) = 1$ PERIODICO T_P

PARI



$$S_K = \frac{1}{T_P}\int_0^{T_P} s(t)\, e^{-jK\omega_0 t}\, dt \qquad \omega_0 = \frac{2\pi}{T_P}$$



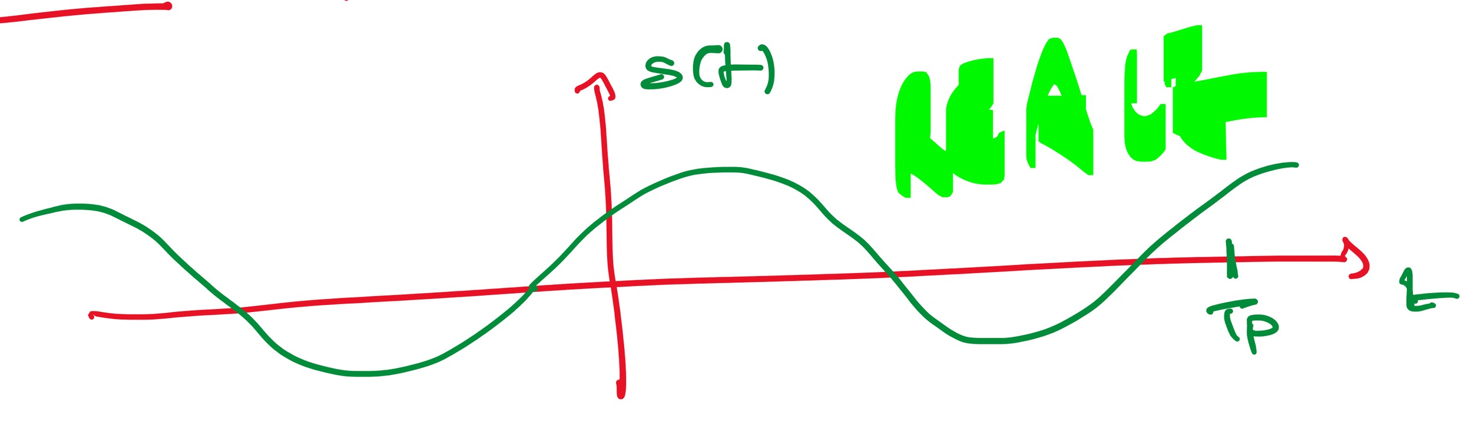
$$= \begin{cases} 0 & K\neq 0 \\ 1 & K=0 \end{cases}$$

$$= \delta(K)$$ PARI

ES 4

S_K PER $s(t) = \cos(\omega_0 t + \varphi_0)$ con $\omega_0 = \frac{2\pi}{T_P}$

REALE



$$S_K = \frac{1}{T_P}\int_0^{T_P} \cos(\omega_0 t + \varphi_0)\, e^{-jK\omega_0 t}\, dt$$

$= \ldots$

$$s(t) = \frac{1}{2}e^{j\varphi_0}\, e^{j\omega_0 t} + \frac{1}{2}e^{-j\varphi_0}\, e^{-j\omega_0 t}$$

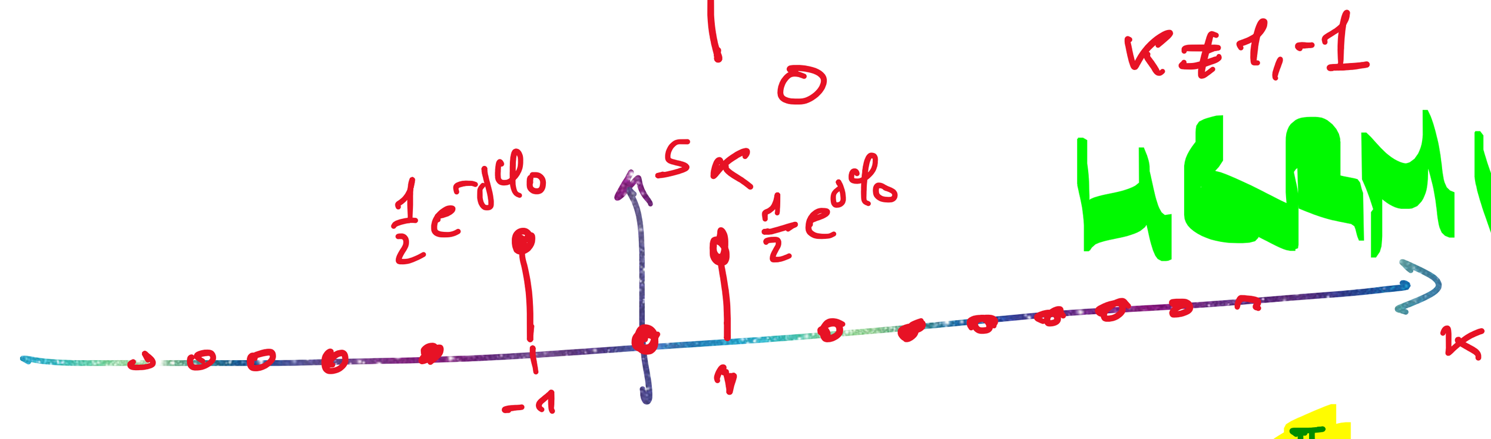
$K=1 \qquad K=-1$

PER ISPEZIONE

$$= \sum_{K=-\infty}^{+\infty} S_K\, e^{jK\omega_0 t}$$

$$S_K = \begin{cases} \frac{1}{2}e^{j\varphi_0} & K=1 \\ \frac{1}{2}e^{-j\varphi_0} & K=-1 \\ 0 & K\neq 1,-1 \end{cases}$$

HERMIT.



ES 5

S_K PER $s(t) = 3 - \sin(2t) + 4\cos(2t) + 2\cos(6t - \pi/4)$

$2\pi\cdot\frac{1}{\pi}t \to \pi \qquad \pi$

$2\pi\cdot\frac{3}{\pi}t \to \pi/3$

REALE

$T_P = \pi$

$\omega_0 = \frac{2\pi}{T_P} = 2$

$$s(t) = 3 - \frac{(e^{j2t} - e^{-j2t})}{2j} + 4\frac{(e^{j2t} + e^{-j2t})}{2}$$

$$+ 2\cdot\frac{(e^{j(6t-\pi/4)} + e^{-j(6t-\pi/4)})}{2}$$

$$= 3 + \left(\frac{-1}{2j} + \frac{4}{2}\right)e^{j2t} + \left(\frac{1}{2j} + \frac{4}{2}\right)e^{-j2t}$$

$K=0 \qquad K=1 \qquad K=-1$

$\omega_0 = 2$

$2 = 1\cdot\omega_0$

$$+ \left(\frac{2}{2}e^{-j\pi/4}\right)e^{j6t} + \left(\frac{2}{2}e^{j\pi/4}\right)e^{-j6t}$$

$K=3 \qquad K=-3$

HERMIT.