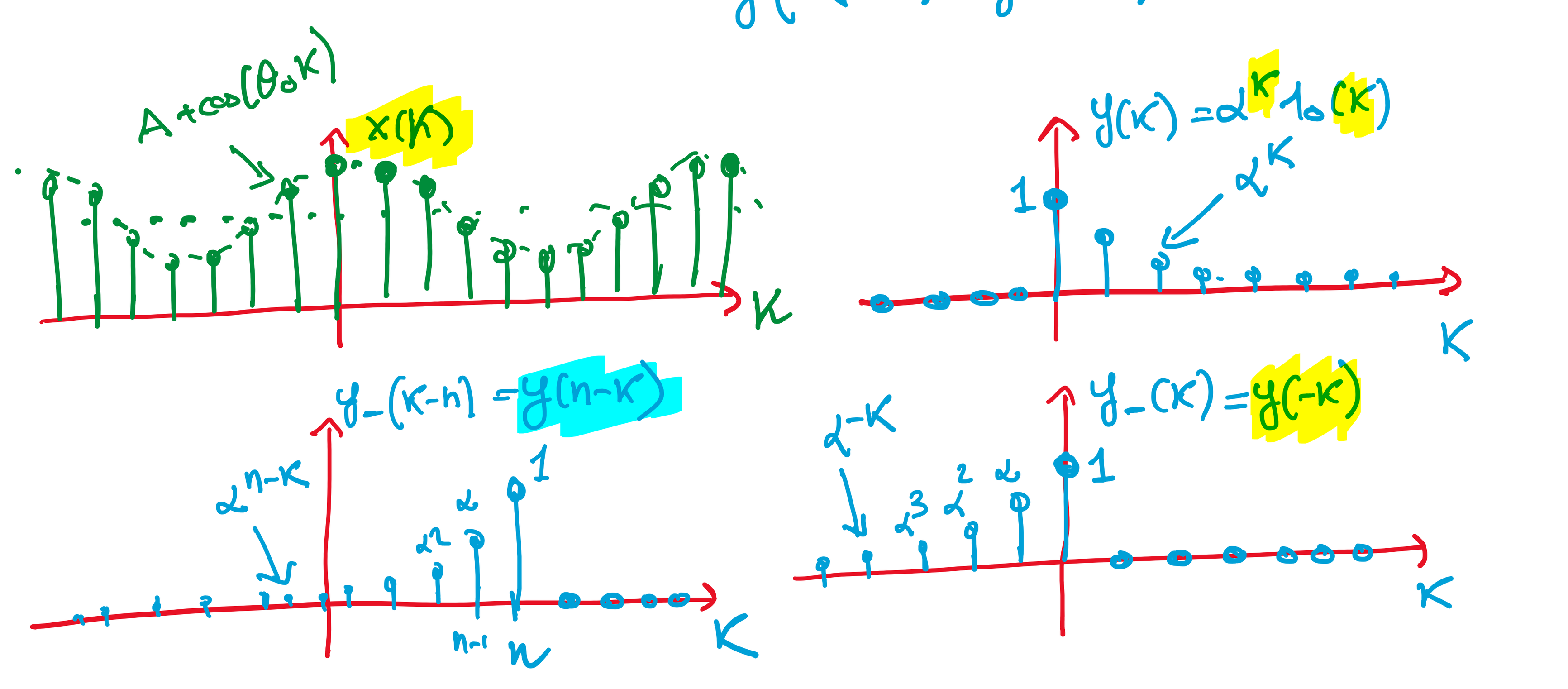
Le09

Friday, 13 March 2026 08:35

ES1 CALCOLARE $z(n)=x*y(n)$ PER $x(n)=A+\cos(\theta_0 n)$

$y(n)=\alpha^n 1_0(n)$ α REALE $|\alpha|<1$

$$z(n)=\sum_{k=-\infty}^{+\infty} x(k)\underbrace{y(n-k)}_{y(-(k-n))=y_-(k-n)}$$



$$z(n)=\sum_{K=-\infty}^{\cancel{+\infty}\ n} \cancel{x(k)}\,\cancel{y(n-k)}\ (A+\cos(\theta_0 K))\,\alpha^{n-K}$$

$$=\sum_{K=-\infty}^{n}(A+\underbrace{\cos(\theta_0 K)}_{\text{EULERO}})\,\alpha^{n-K}$$

$$\frac{1}{2}e^{j\theta_0 K}+\frac{1}{2}e^{-j\theta_0 K}$$

$$=\sum_{K=-\infty}^{n} A\alpha^{n-K}+\frac{1}{2}e^{j\theta_0 K}\alpha^{n-K}+\frac{1}{2}e^{-j\theta_0 K}\alpha^{n-K}$$

$m=n-K$

$$=\sum_{m=0}^{+\infty} A\alpha^m+\frac{1}{2}e^{j\theta_0(n-m)}\alpha^m+\frac{1}{2}e^{-j\theta_0(n-m)}\alpha^m$$

$$=A\sum_{m=0}^{+\infty}\alpha^m+\frac{1}{2}e^{j\theta_0 n}\sum_{m=0}^{\infty}e^{-j\theta_0 m}\alpha^m \quad (\alpha e^{-j\theta_0})^m$$

$$+\frac{1}{2}e^{-j\theta_0 n}\sum_{m=0}^{\infty}e^{j\theta_0 m}\alpha^m \quad (\alpha e^{j\theta_0})^m$$

$$=\frac{A}{1-\alpha}+\frac{1}{2}\frac{e^{j\theta_0 n}}{\underbrace{1-\alpha e^{-j\theta_0}}_{a}}+\frac{1}{2}\frac{e^{-j\theta_0 n}}{\underbrace{1-\alpha e^{j\theta_0}}_{a^*}}$$

$$\boxed{z(n)=\frac{A}{1-\alpha}+\mathrm{Re}\left[\frac{e^{j\theta_0 n}}{1-\alpha e^{-j\theta_0}}\right]}$$

$$B=1-\alpha e^{-j\theta_0}=|B|e^{j\varphi_B}$$

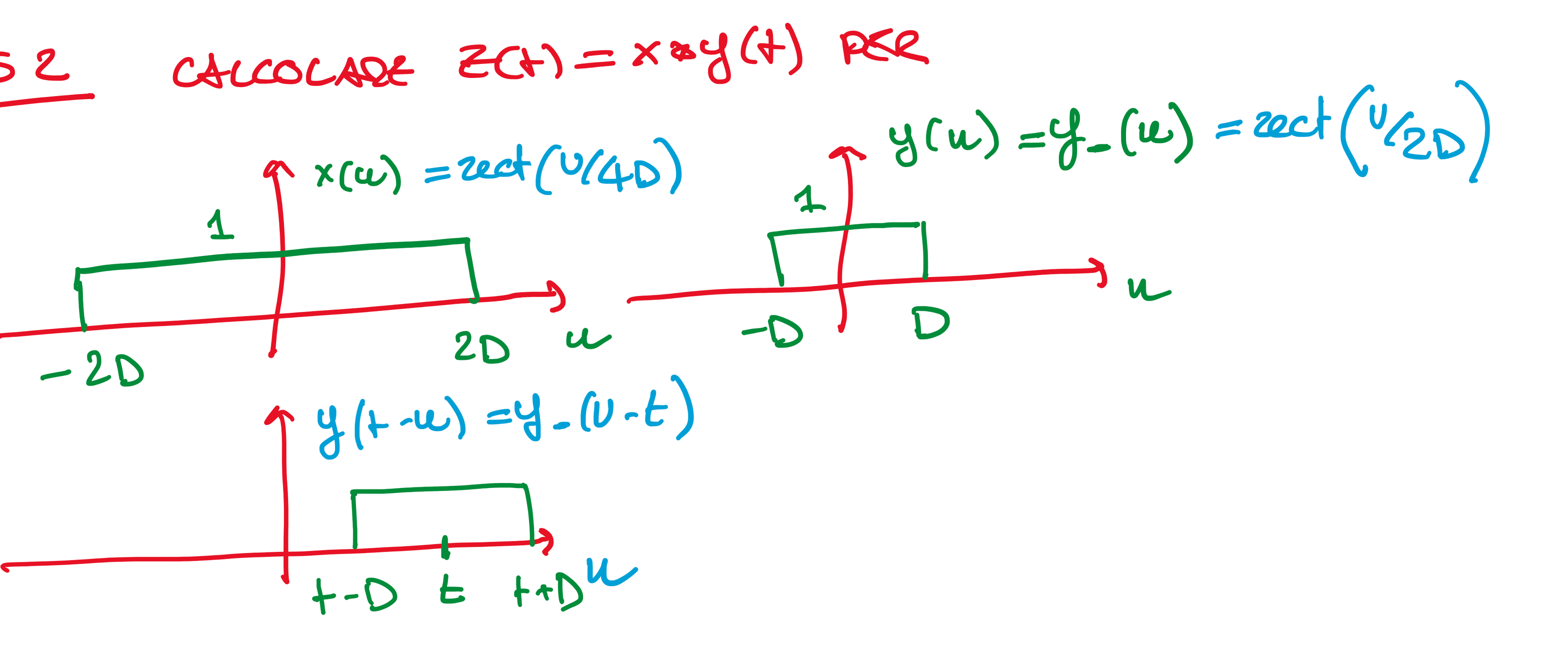
$$\mathrm{Re}\left[\frac{e^{j\theta_0 n}}{|B|\cancel{e^{j\varphi_B}}}\frac{e^{-j\varphi_B}}{\cancel{e^{-j\varphi_B}}}\right]=\mathrm{Re}\left[\frac{e^{j(\theta_0 n-\varphi_B)}}{|B|}\right]$$

$$=\mathrm{Re}\left[\frac{\cos(\theta_0 n-\varphi_B)}{|B|}+j\frac{\sin(\theta_0 n-\varphi_B)}{|B|}\right]$$

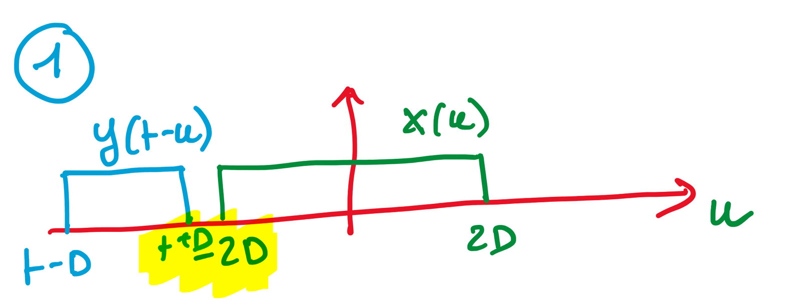
$$=\frac{\cos(\theta_0 n-\varphi_B)}{|B|}$$

$$\boxed{z(n)=\frac{A}{1-\alpha}+\frac{\cos(\theta_0 n-\varphi_B)}{|B|},\quad B=1-\alpha e^{-j\theta_0}}$$

ES2 CALCOLARE $z(t)=x*y(t)$ PER



1

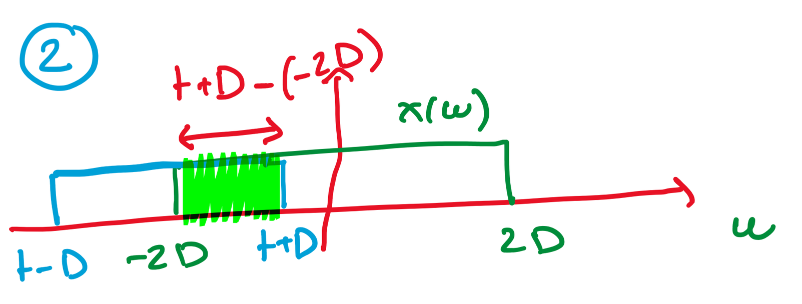


$t+D<-2D$

OVVERO

$t<-3D \rightarrow z(t)=0$

2

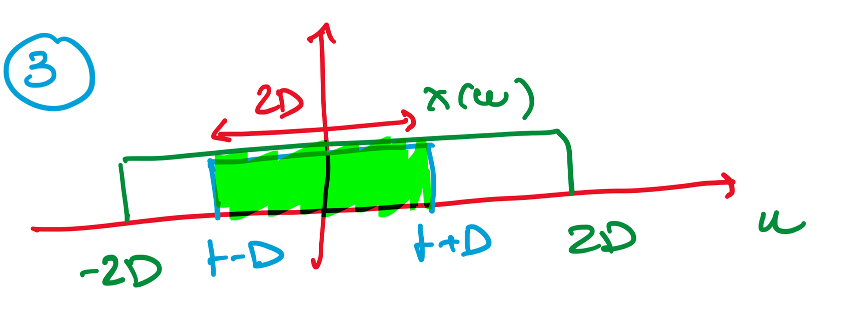


$t-D<-2D<t+D$

OVVERO

$-3D<t<-D \rightarrow z(t)=t+3D$

3

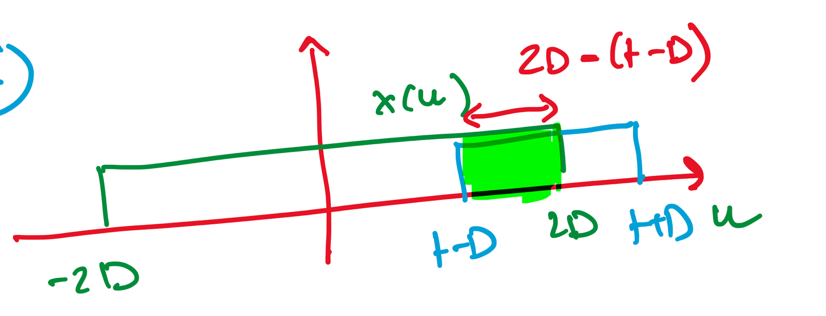


$-2D<t-D<t+D<2D$

OVVERO

$-D<t<D \rightarrow z(t)=2D$

4

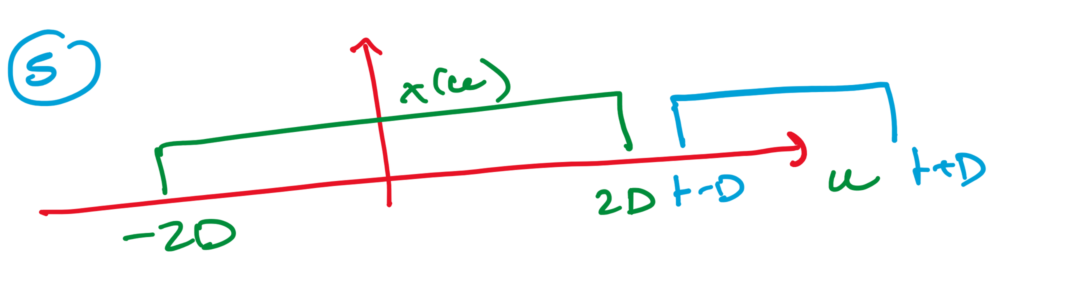


$t-D<2D<t+D$

OVVERO

$D<t<3D \rightarrow z(t)=3D-t$

5

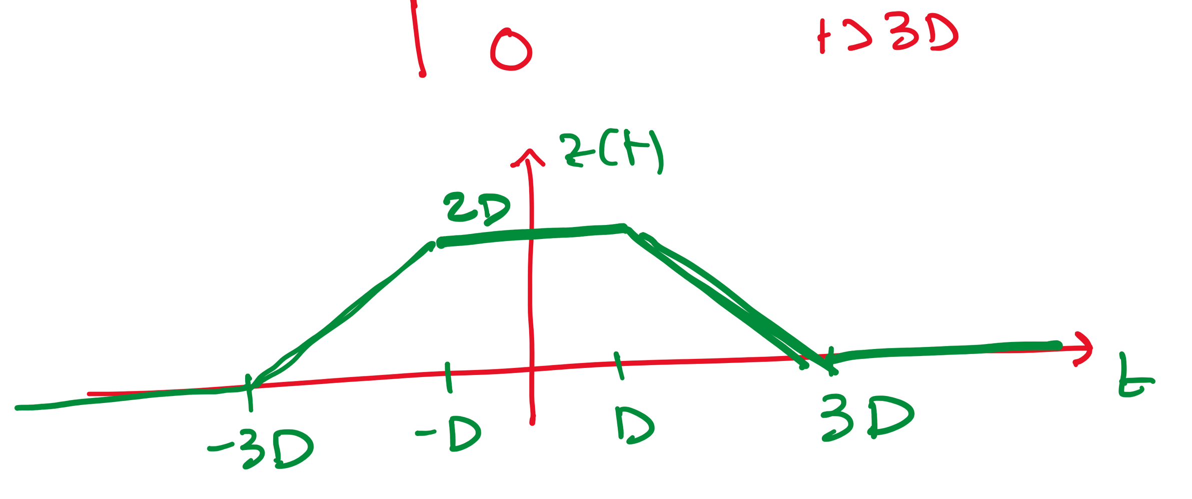


$t-D>2D$

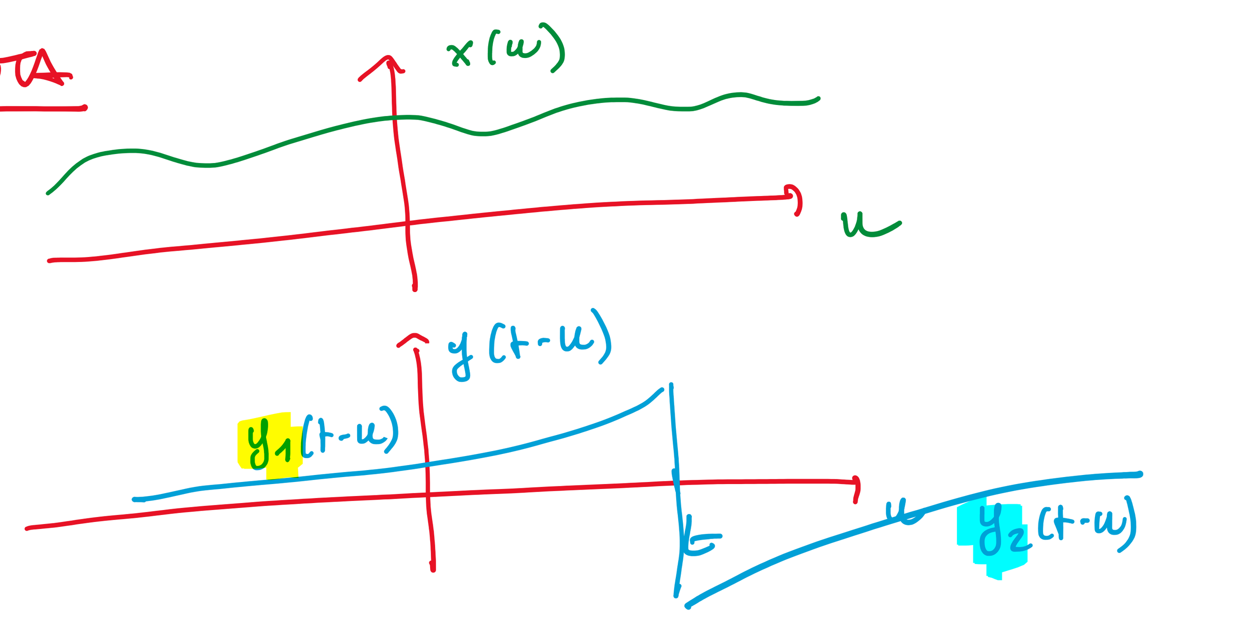
OVVERO

$t>3D \rightarrow z(t)=0$

$$z(t)=\begin{cases}0 & t<-3D\\ t+3D & -3D<t<-D\\ 2D & -D<t<D\\ 3D-t & D<t<3D\\ 0 & t>3D\end{cases}$$



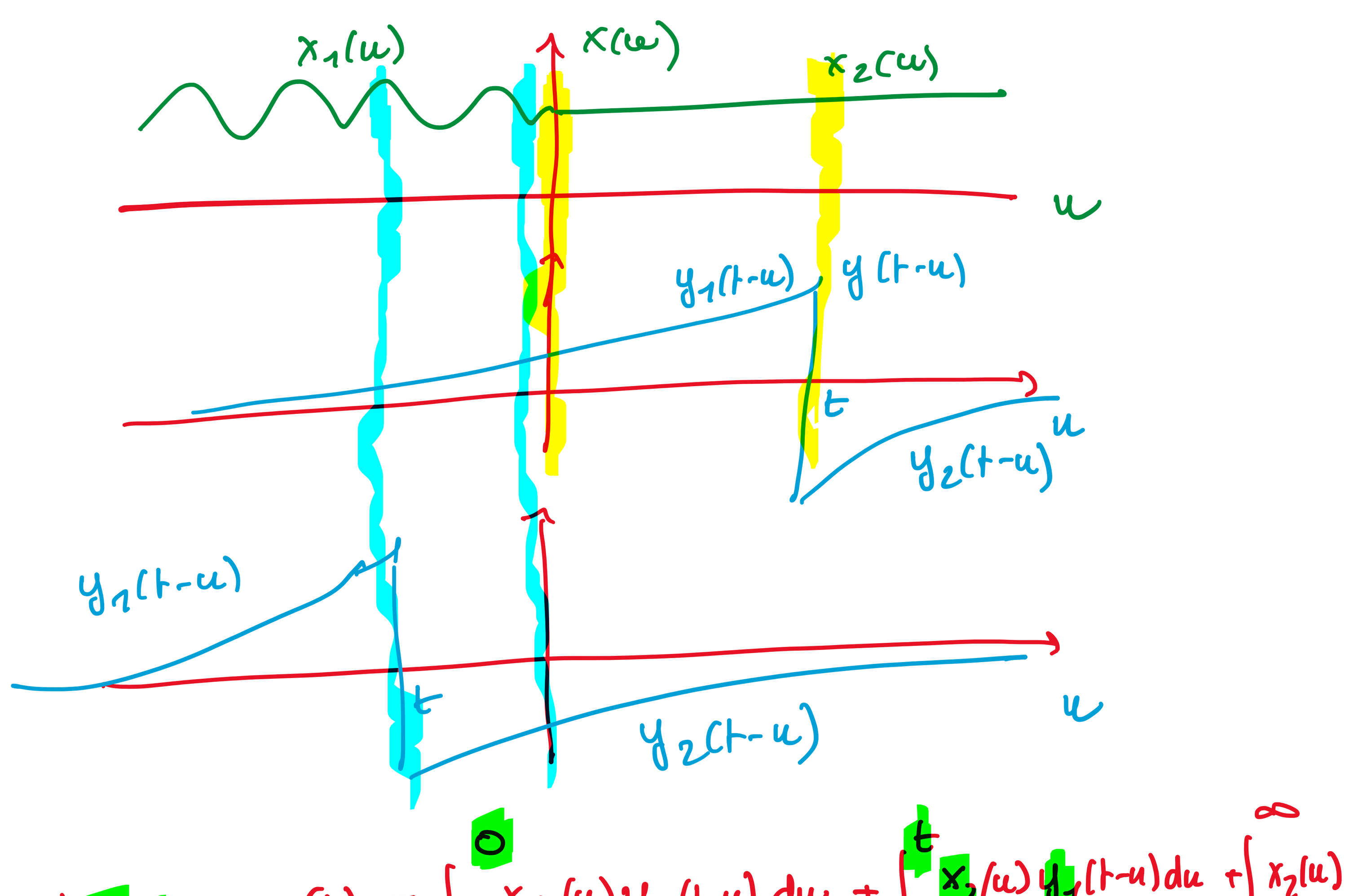
NOTA



$$z(t)=\int_{-\infty}^{+\infty}x(u)\,y(t-u)\,du=\int_{-\infty}^{t}x(u)\,y_1(t-u)\,du+\int_{t}^{+\infty}x(u)\,y_2(t-u)\,du$$

1 REGIONE x $z(t)$

2 INTEGRAZIONI



$t>0$ $\quad z(t)=\int_{-\infty}^{0}x_1(u)\,y_1(t-u)\,du+\int_{0}^{t}x_2(u)\,y_1(t-u)\,du+\int_{t}^{\infty}x_2(u)\,y_2(t-u)\,du$

$t<0$ $\quad z(t)=\int_{-\infty}^{t}x_1(u)\,y_1(t-u)\,du+\int_{t}^{0}x_1(u)\,y_2(t-u)\,du+\int_{0}^{\infty}x_2(u)\,y_2(t-u)\,du$

2 REGIONI

3 INTEGRALI