Le04

Monday, 2 March 2026 08:30

$s(t) = A \longrightarrow$ $m_s = A$, $P_s = |A|^2$

$s(t) = A\cos(2\pi f_0 t + \varphi_0) \longrightarrow$ $m_s = 0$, $P_s = \frac{A^2}{2}$

$f_0 \neq 0$

$s(t) = A e^{j2\pi f_0 t} \longrightarrow$ $m_s = 0$, $P_s = |A|^2$

$f_0 \neq 0$

$s(t) = \sum_{k=1}^{K} A_k e^{j2\pi f_k t} + A_0 \longrightarrow$ $m_s = 0 + A_0$, $P_s = \sum_{k=1}^{K} |A_k|^2 + |A_0|^2$

$f_k \neq 0$

f_k TUTTI DIVERSI

$s(t) = \sum_{k=1}^{K} A_k \cos(2\pi f_k t + \varphi_k) + A_0 \longrightarrow$ $m_s = 0 + A_0$, $P_s = \sum_{k=1}^{K} \frac{A_k^2}{2} + A_0^2$

$f_k \neq 0$

$f_k > 0$ f_k TUTTI DIVERSI

CASO ~~PARTICOLARE~~ IN CUI LA SOMMA HA COME POTENZA LA SOMMA DELLE POTENZE

CASO GENERALE

$s(t) = x(t) + y(t)$

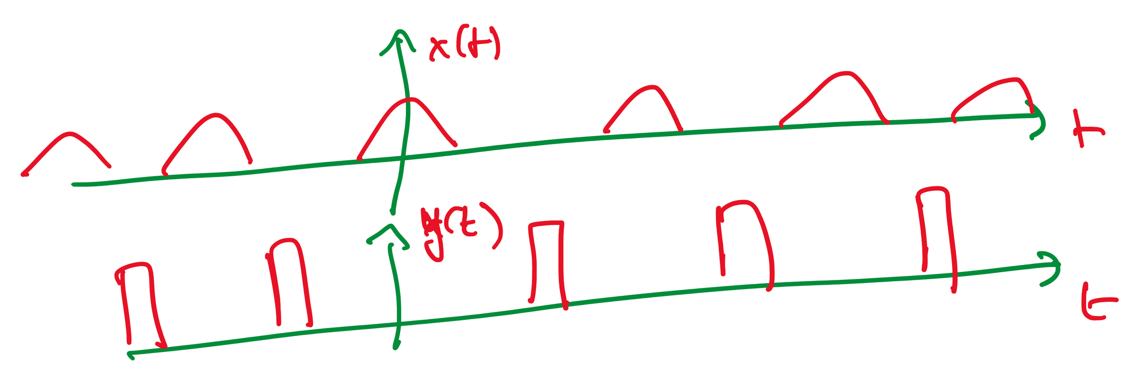
$s^2(t) = x^2(t) + y^2(t) + 2x(t)y(t)$

$P_s = P_x + P_y + 2P_{xy}$

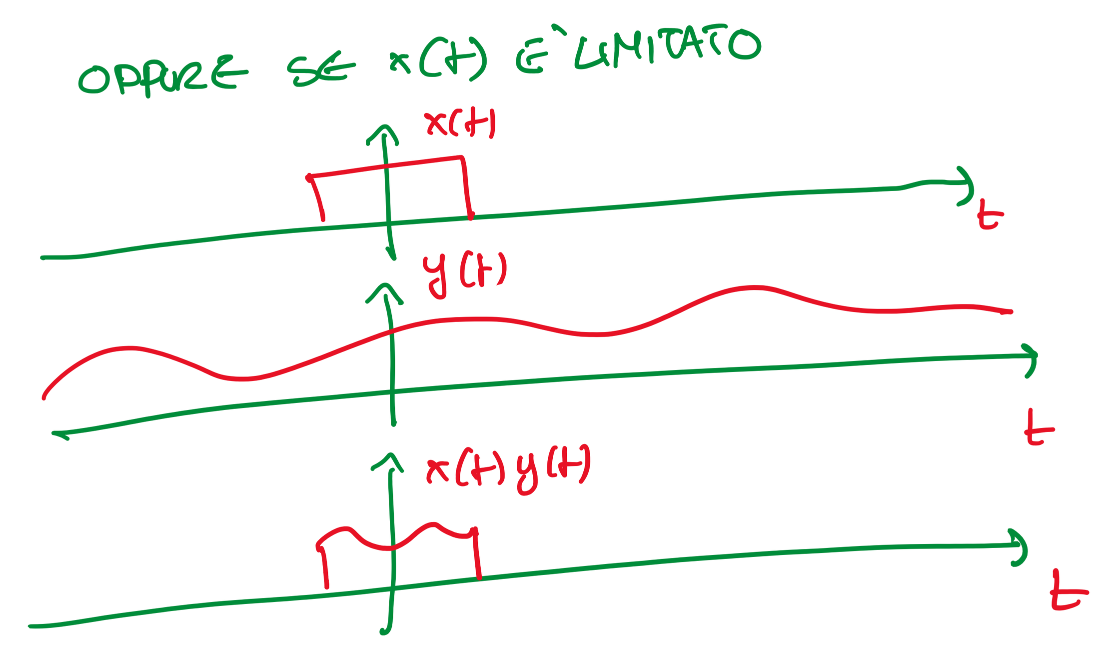
CONTRIBUTO = 0 SOLO SE LA MEDIA DI $x(t)y(t)$ È UGUALE A 0

AD. ESEMPIO

$P_{xy} = 0$ SE x E y SONO ATTIVI IN TEMPI DIVERSI



OPPURE SE $x(t)$ È LIMITATO



ES1 CAMPIONAMENTO SINUSOIDI

$f_0 T = \frac{K}{N}$

$s(n) = \cos\left(\frac{\pi}{16} n\right) = \cos(\theta_0 n) = \cos(2\pi f_0 T n)$

$\theta_0 = \pi/16$ $\quad f_0 T = \frac{\theta_0}{2\pi}$

$\frac{\theta_0}{2\pi} = \frac{\pi}{16 \cdot 2\pi} = \frac{1}{32} = \frac{K}{N}$

$N = 32$ PERIODO

$s(n) = \sin\left(\frac{7}{5}\pi n\right) \longrightarrow \frac{\theta_0}{2\pi} = \frac{7\pi}{5} \cdot \frac{1}{2\pi} = \frac{7}{10}$

$N = 10$ PERIODICO

$s(n) = \cos(2n) \longrightarrow \frac{\theta_0}{2\pi} = \frac{2}{2\pi} = \frac{1}{\pi}$ NON RAZIONALE

NON PERIODICO

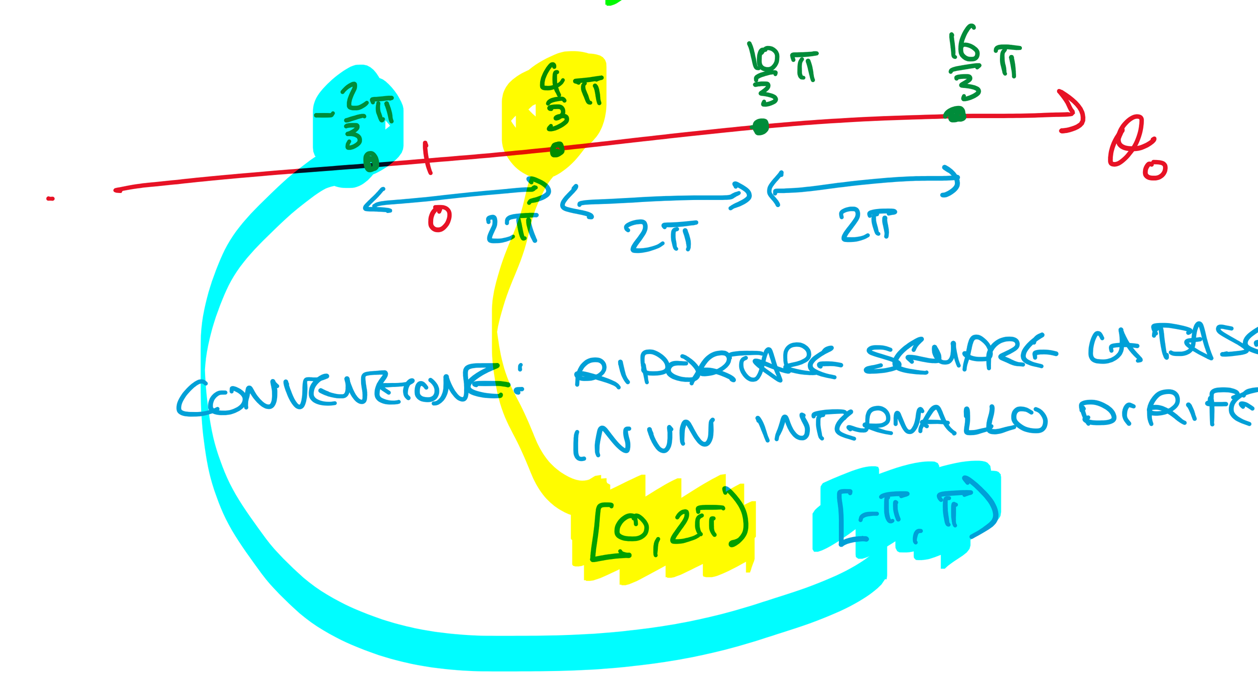
ESEMPIO $s(n) = \cos\left(\frac{16}{3}\pi n\right) = \cos\left(\frac{16}{3}\pi n - 2\pi n\right)$

$= \cos\left(\frac{10}{3}\pi n\right) = \cos\left(\frac{10}{3}\pi n - 2\pi n\right)$

$= \cos\left(\frac{4}{3}\pi n\right) = \cos\left(\frac{4}{3}\pi n - 2\pi n\right)$

$= \cos\left(-\frac{2}{3}\pi n\right)$

$= \cos\left(\frac{2}{3}\pi n\right)$



CONVENZIONE: RIPORTARE SEMPRE LA FASE IN UN INTERVALLO DI RIFERIMENTO

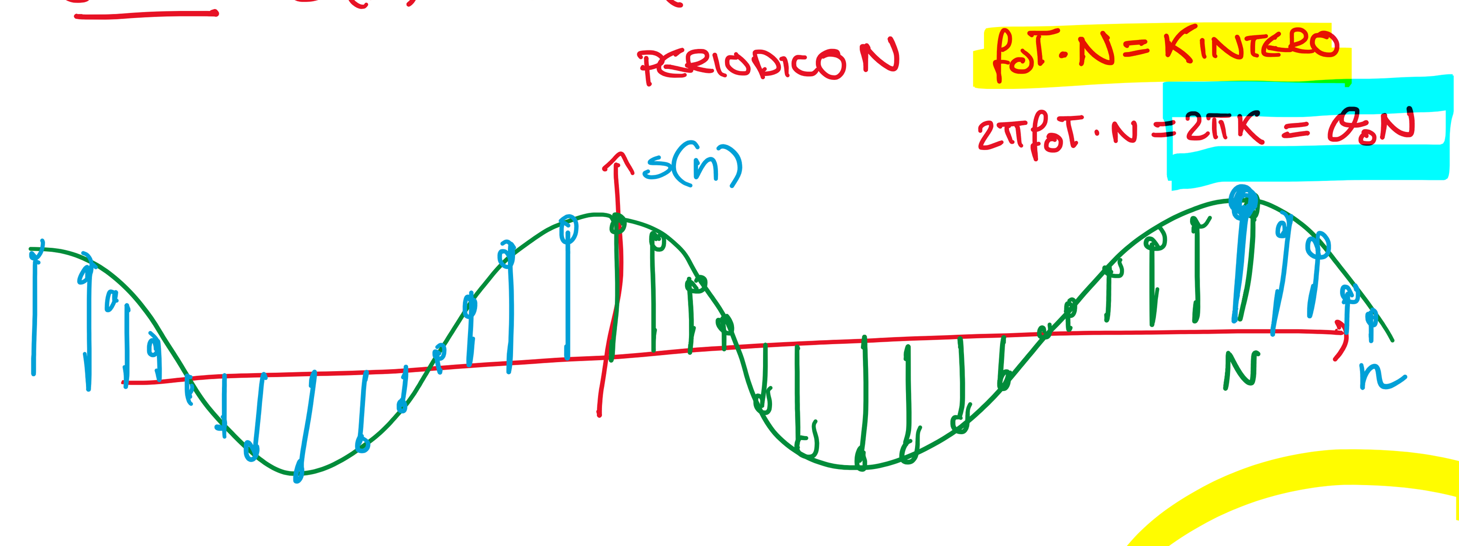
$[0, 2\pi)$ $\quad [-\pi, \pi)$

ES2

$s(n) = A\cos(2\pi f_0 n T) = A\cos(\theta_0 n)$

PERIODICO N $\quad f_0 T \cdot N = K$ INTERO

$2\pi f_0 T \cdot N = 2\pi K = \theta_0 N$



$A_s(N) = \sum_{n=0}^{N-1} A\cos(\theta_0 n)$

EULERO

$\cos\alpha = \frac{e^{j\alpha} + e^{-j\alpha}}{2}$

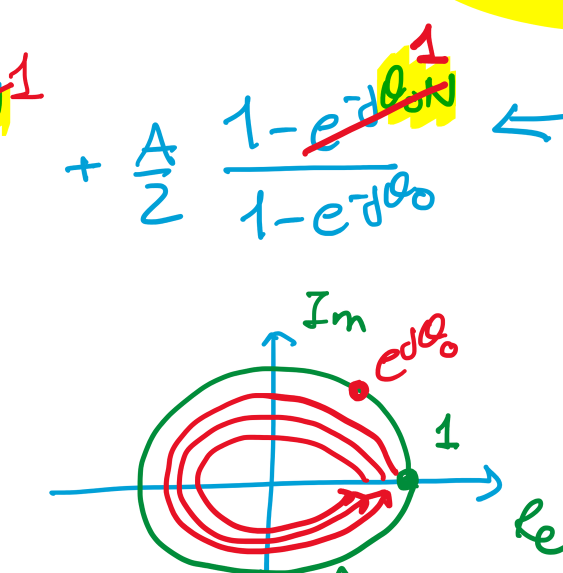
$\sin\alpha = \frac{e^{j\alpha} - e^{-j\alpha}}{2j}$

$= \sum_{n=0}^{N-1} \frac{A}{2}\left(e^{j\theta_0}\right)^n + \frac{A}{2}\left(e^{-j\theta_0}\right)^n$

$= \frac{A}{2} \frac{1 - \cancel{e^{j\theta_0 N}}^{\,1}}{1 - e^{j\theta_0}} + \frac{A}{2} \frac{1 - \cancel{e^{-j\theta_0 N}}^{\,1}}{1 - e^{-j\theta_0}} \longleftarrow \theta_0 N = 2\pi K$

$A_s(N) = 0$

$\underbrace{\left(e^{j2\pi}\right)^K}_{1^K} = 1$



$m_s = 0$

$|s(n)|^2 = A^2 \cos^2(\theta_0 n) = \frac{A^2}{2} + \frac{A^2}{2}\cos(2\theta_0 n)$

$P_s = \frac{A^2}{2}$ CASO IN CUI $2\theta_0 \neq 2\pi K$

ES3

$s(n) = Ae^{j\theta_0 n}$ con θ_0 generico e $\theta_0 \neq 2\pi K$

ASSICURA CHE IL SEGNALE NON SIA COSTANTE

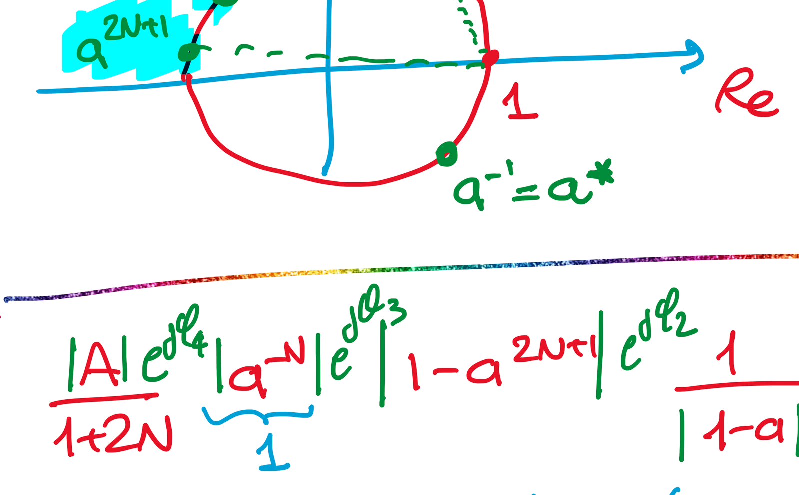
$\Downarrow$

NON È DETTO CHE $s(n)$ SIA PERIODICO

$m_s = \lim_{N\to\infty} \frac{1}{1+2N} \cdot \sum_{n=-N}^{N} A \underbrace{\left(e^{j\theta_0}\right)^n}_{a^n}$ $\quad a = e^{j\theta_0}$

$= \lim_{N\to\infty} \frac{Aa^{-N}}{1+2N} \sum_{m=0}^{2N} a^{m \cancel{-N}}$

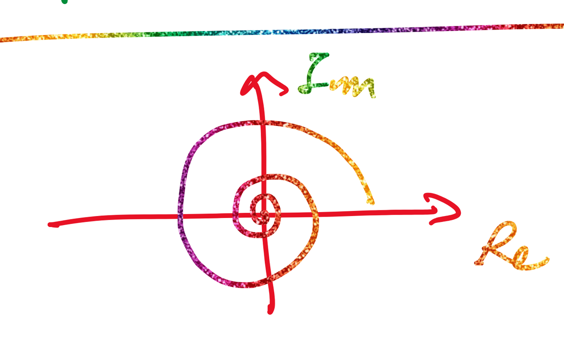
$= \lim_{N\to\infty} \frac{A a^{-N}}{1+2N} \cdot \frac{1 - a^{2N+1}}{1 - a} = 0$



$m_s = 0$

$\frac{|A| e^{j\varphi_4}}{1+2N} \underbrace{|a^{-N}|}_{1} e^{j\varphi_3} \left|1 - a^{2N+1}\right| e^{j\varphi_2} \frac{1}{|1-a| e^{j\varphi_1}}$

$= \frac{|A|}{1+2N} \frac{|1 - a^{2N+1}|}{|1-a|} \cdot \frac{\cancel{e^{j(\varphi_4 + \varphi_3 + \varphi_2 - \varphi_1)}}}{e^{j\theta_S}}$



$|s(n)|^2 = |A|^2$

$P_s = |A|^2$