

Note: Texts with color ~~blue~~ is the correct exercise

Text with color ~~blue~~ are additional notes and my attempts at solving the exercise

Text with color ~~red~~ are corrections of my mistakes or remarks

1. $L_1 = \{a^n b^n \mid n \geq 1\}$ we've already seen not reg. lang.

CFG variables = $\{S\}$

CFG terminals = $\{a, b\}$

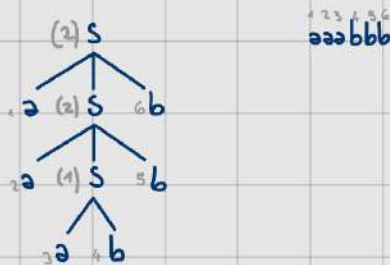
now let's write rules to generate the $w \in L_1$

CFG rules: 1. $S \rightarrow ab$

2. $S \rightarrow aSb$

← I can generate in the middle of the string, we don't need to count to be sure $w \in L_1$

Let's see example



FA can't work here, you have finite number of states and needs to go left to right

2. $L_2 = \{a^n b^{2n} \mid n \geq 1\}$

$T = \{a, b\}$ $V = \{S\}$

rules $\rightarrow S \rightarrow abb$

$S \rightarrow aSbb$

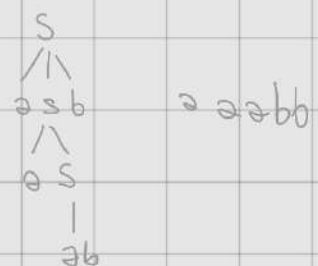
3. $L_3 = \{a^n b^m \mid n \geq m \geq 1\}$

$T = \{a, b\}$ $V = \{S, A, B\}$

rules $\rightarrow S \rightarrow ab$ $\rightarrow$ identical to L_1

$S \rightarrow aSb$

$S \rightarrow aS$ Can generate "a" without "b"



4. $L_4 = \{a^n b^m \mid n, m \geq 1\}$

⚠ S is the initial symbol

1. $S \rightarrow AB$ ← you always start from S

2. $A \rightarrow a$

4. $B \rightarrow b$

A generates block of a's, B the b's

3. $A \rightarrow aA$

5. $B \rightarrow bB$

Or $S \rightarrow ab$ also works

$S \rightarrow aS$

$S \rightarrow Sb$

Ex.

1. $L_2 = \{w \mid w = a^i b^j c^k \mid i, j, k \geq 1\}$

$S \rightarrow ABC$

$A \rightarrow \dots$

$B \rightarrow \dots$

$C \rightarrow c, C \rightarrow Cc$

2. $L_3 = \{w \mid w = ba^n ba^n b, n \geq 1\}$

$S \rightarrow bAbAb$ (2A different, independent e.g. $bAbAb \Rightarrow babbAb \Rightarrow babbAab \Rightarrow babbbaab$)

$A \rightarrow a \quad A \rightarrow Aa$

$S \rightarrow bAb$

$A \rightarrow aAa$

$A \rightarrow aba \quad A \rightarrow b$ wrong! since $n \geq 1$

$S \Rightarrow bAb \Rightarrow babbab$

$$L_1 = \{w \mid w = x2x^R, x \in \{0,1\}^*\}$$

$$\Sigma = \{0, 1, 2\} \quad \text{odd length}$$

$$2 \text{ only } 020, 121$$

$$S \rightarrow 2$$

$$S \rightarrow 0s0 \mid 1s1$$

$$L_2 = \{w \mid a^i b^j c^k, i, j, k \geq 1, j \neq k\}$$

$$S \rightarrow AB$$

$$A \rightarrow a \mid Aa$$

$$B \rightarrow bbc \mid bac \mid \cancel{bcc}$$

can't have
bcccc

$$A \rightarrow a \mid Aa$$

$b^j c^k$ as a "single block"

$$F \rightarrow bFc$$

$$F \rightarrow F_1 \mid F_2 \quad F_1 \rightarrow b \mid F_1 b$$

$$F_2 \rightarrow c \mid F_2 c$$

$$S \rightarrow AF$$

Ex. 1

$$L = \{a^i b^j c^k \mid i, j, k \geq 1, k = i + j\}$$

Shortest string is 4 : abcc $i=1, j=1, k=1+1=2$

Not regular (can't do sums nor grammar)

Context-free grammar

Falso io

$$S_1 \rightarrow a S_1 c \mid a S_2 c$$

$$S_2 \rightarrow b S_2 c \mid bc \mid \lambda \quad j \geq 1$$

Let's see $w \in L$: $aaabbbbccccccc$ X



$$1. S \rightarrow aSc \mid aBc$$

$$3. B \rightarrow bBc \mid bc$$

2. Stop generating "a...c" and start with "bc"

