Simulation and control of a magnetic levitation system

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Reporting instructions

- max 3 persons per group (only one shall submit, though)
- the report consists of a `NUMERIMATRICOLA.pdf` document + a `NUMERIMATRICOLA.mlx` file
- both files shall be sent to `niccolo.turcato@phd.unipd.it` before the end of June 22nd
- the `.pdf` shall have one section per assignment. You may find a `template-report.tex` template at `https://www.overleaf.com/read/dfvtyjyzrnqv#20635d`; you may also use some wysiwyg editor though
- the `.mlx` shall be a completion of the template available in the "Matlab" folder at the same url

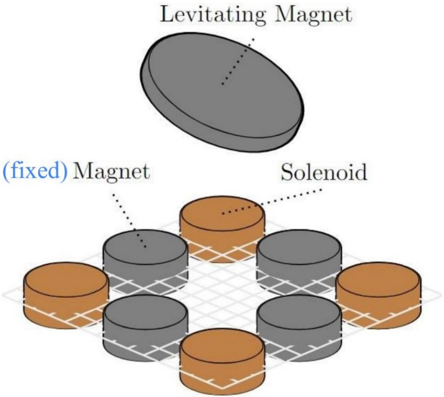
Information about the grading process

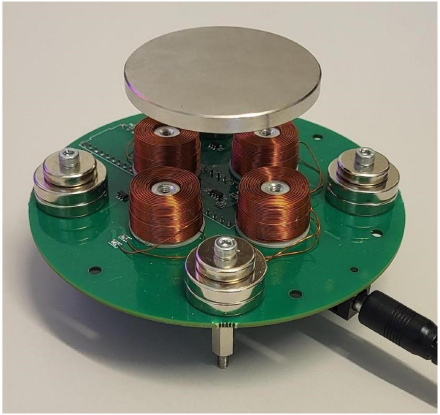
- everybody in the group will get the same grade
- grades will reflect completeness and correctedness of the submitted solutions in a per-assignment fashion (i.e., each assignment has its grade)
- the grades will be directly incorporated in the one for the final exam (thus contribute to the "18")
- these grades shall thus be a sort of "bonus" valid until the course will be given next year

Suggestions

- use this occasion to learn LaTeX (a nice video-introduction to LaTeX is at `https://www.youtube.com/watch?v=zqQM66uAig0`)
- consider also using `overleaf.com` and `mathpix.com/`
- if you can't Matlab, we suggest `https://matlabacademy.mathworks.com/details/matlab-onramp/gettingstarted`
- if you can't Simulink, we suggest `https://matlabacademy.mathworks.com/details/simulink-onramp/simulink`
- ask chatGPT (or whatever you prefer) for pieces of code (both Matlab and LaTeX)

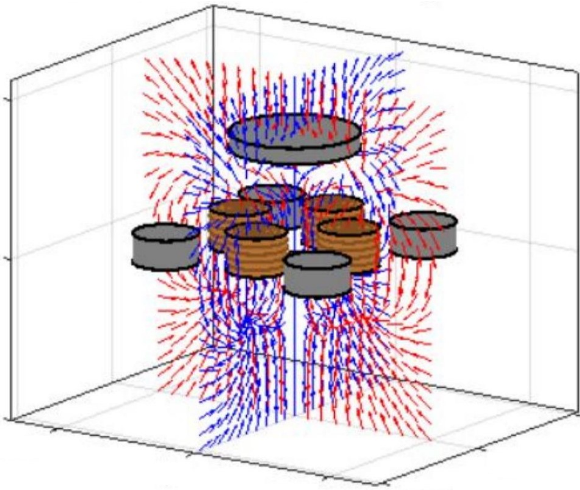
The system





Characteristics

- actuated by controlling the solenoids currents
- has one equilibrium (denoted by $\tilde{\star}$)
- lack of friction makes it not BIBO stable



Suggested literature

Modeling and Control of a Magnetic Levitation Platform

Hans Alvar Engmark * **Kiet Tuan Hoang** *

* *Department of Engineering Cybernetics, The Norwegian University of*

$$\dot{\boldsymbol{x}} = \mathbf{A}\boldsymbol{x} + \mathbf{B}f(\boldsymbol{x}, \boldsymbol{u}) \quad (11a)$$

$$\boldsymbol{y} = \mathbf{C}\boldsymbol{x} \quad (11b)$$

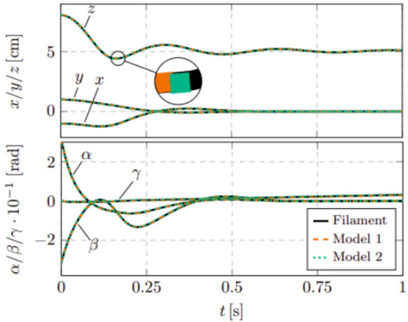
where

$$\mathbf{A} = \begin{bmatrix} \mathbf{0}_{6\times6} & \mathbf{I}_6 \\ \mathbf{0}_{6\times6} & \mathbf{0}_{6\times6} \end{bmatrix} \quad (11c)$$

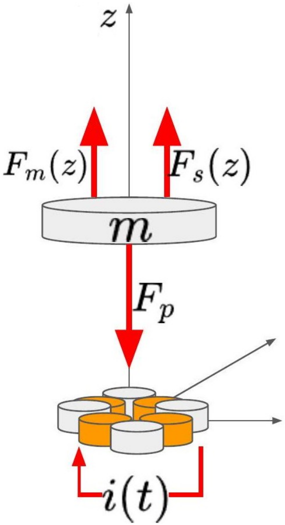
$$\mathbf{B} = \begin{bmatrix} \mathbf{0}_{6\times6} \\ \mathbf{I}_6 \end{bmatrix} \quad (11d)$$

and

$$f(\boldsymbol{x}, \boldsymbol{u}) = \begin{bmatrix} m\mathbf{I}_3 & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathcal{I} \end{bmatrix}^{-1} \left(\begin{bmatrix} \boldsymbol{F}(\boldsymbol{x}, \boldsymbol{u}) \\ \boldsymbol{\tau}(\boldsymbol{x}, \boldsymbol{u}) \end{bmatrix} - \begin{bmatrix} \boldsymbol{F}_g \\ \boldsymbol{w} \times \mathcal{I}\boldsymbol{w} \end{bmatrix} \right) \quad (11e)$$



Simplifying the original model to get a SISO one (Single input - single output)



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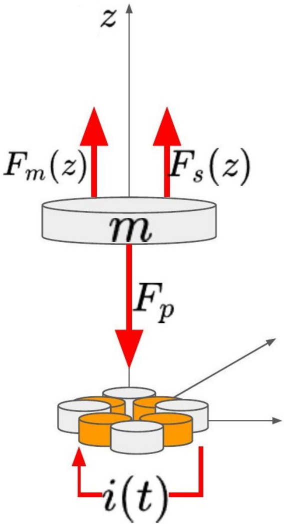
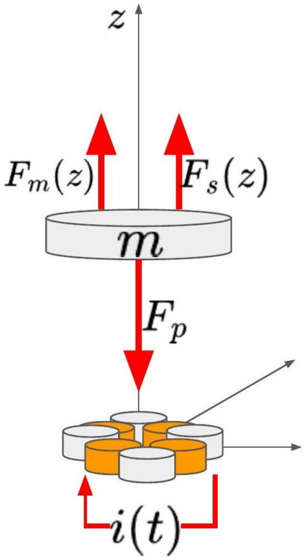


Assumptions on the levitating magnet:

- mass = approximated with m
- movement = only in the vertical direction, and disregarding rotations
- moment of inertia = negligible
- push force from fixed magnets = upwards and inversely proportional with the distance
- push / pull force from solenoids = inversely proportional with the distance and proportional with the input current $i(t)$

Simplified SISO model



$$m\ddot{z} = -F_p + F_m(z) + F_s(z,i) - d\dot{z}$$

with

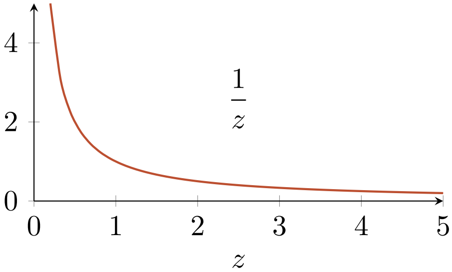
$$F_p = mg$$

$$F_m(z) = \frac{b_m}{z}$$

$$F_s(z,i) = \frac{b_s}{z} i$$

How nonlinear is the system?

$$F_m(z) = \frac{b_m}{z} \qquad F_s(z,i) = \frac{b_s}{z} i$$



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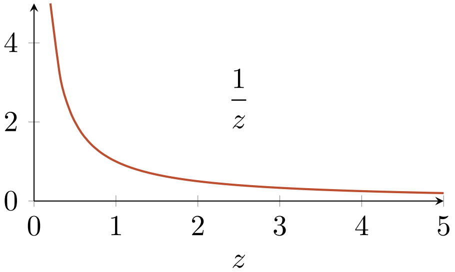
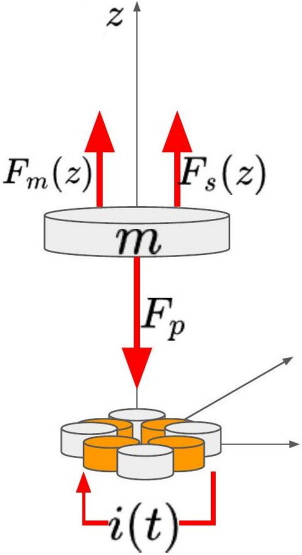


to obtain a LTI model we need to linearize first;
but linearizing requires choosing an equilibrium first

Computing the equilibria of the system

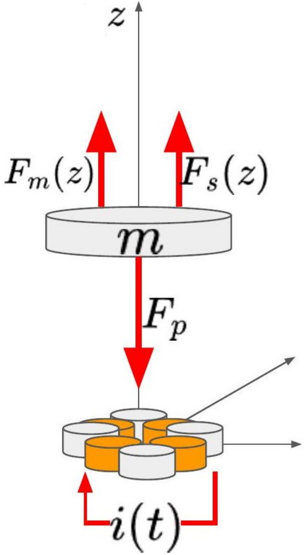


$$m\ddot{z} = -mg + \frac{b_m}{z} + \frac{b_s}{z} i - d\dot{z}$$

implies

$$0 = -mg + \frac{b_m}{z} + \frac{b_s}{z} i - 0$$

Computing the equilibria of the system



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implies

$$0 = -mg + \frac{b_m}{z} + \frac{b_s}{z} i - 0$$

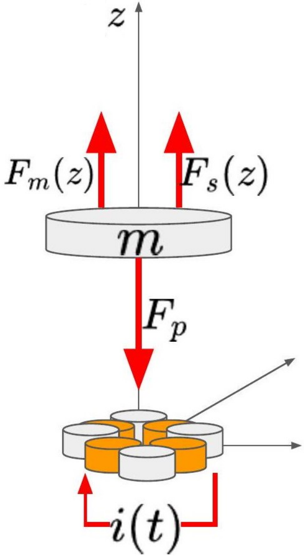
linearizing around (z_0, i_0) then implies the change of variables

$$z = z_0 + \widetilde{z} \qquad i = i_0 + \widetilde{i}$$

and the relation

$$-mg + \frac{b_m}{z_0} + \frac{b_s}{z_0} i_0 = 0 \quad \rightarrow \quad z_0 = \frac{b_m}{mg} + \frac{b_s}{mg} i_0$$

Linearizing $F_m(z) = \dfrac{b_m}{z}$, i.e., the force from the permanent magnets



$$F_m(z) \mapsto F_m(z_0) + \frac{\partial F_m(z_0)}{\partial z}\tilde{z}$$

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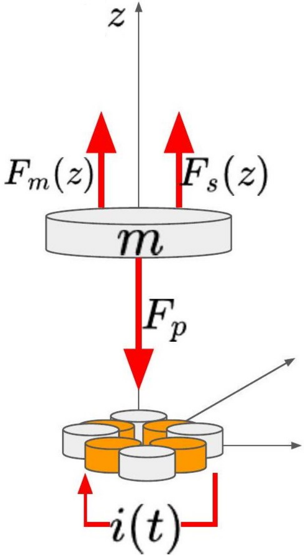
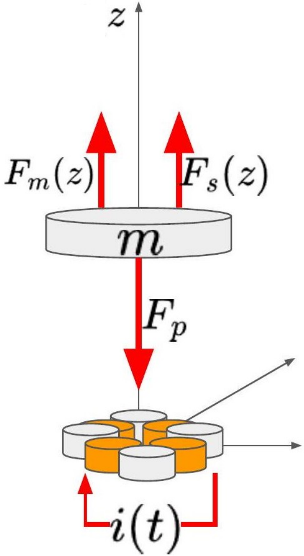


$$F_m(z) \mapsto F_m(z_0) + \frac{\partial F_m(z_0)}{\partial z}\widetilde{z}$$

thus

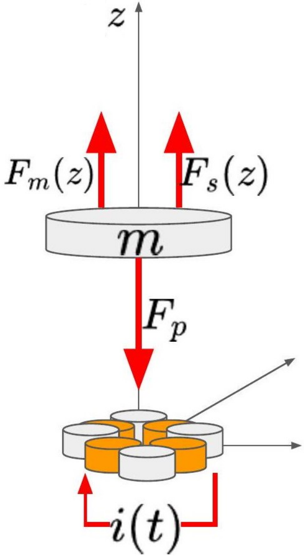
$$\frac{b_m}{z} \mapsto \frac{b_m}{z_0} - \frac{b_m}{z_0^2}\widetilde{z}$$

Linearizing $F_s(z,i) = \frac{b_s}{z} i$, i.e., the force from the solenoids



$$F_s(i,z) \mapsto F_s(i_0,z_0) + \frac{\partial F_s(i_0,z_0)}{\partial i}\tilde{i} + \frac{\partial F_s(i_0,z_0)}{\partial z}\tilde{z}$$

Linearizing $F_s(z,i) = \frac{b_s}{z} i$, i.e., the force from the solenoids



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thus

$$\frac{b_s}{z} i \mapsto \frac{b_s i_0}{z_0} + \frac{b_s}{z_0}\tilde{i} - \frac{b_s i_0}{z_0^2}\tilde{z}$$

Linearization - putting all the ingredients together

first ingredient: $m\ddot{z} = -mg + \frac{b_m}{z} + \frac{b_s}{z} i - d\dot{z}$

second ingredient: $z = z_0 + \widetilde{z} \qquad i = i_0 + \widetilde{i}$

third ingredient: $\frac{b_m}{z} \mapsto \frac{b_m}{z_0} - \frac{b_m}{z_0^2}\widetilde{z} \qquad \frac{b_s}{z} i \mapsto b_s \frac{i_0}{z_0} + \frac{b_s}{z_0}\widetilde{i} - \frac{b_s i_0}{z_0^2}\widetilde{z}$

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fourth ingredient: $i_0 = 0$

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fourth ingredient: $i_0 = 0$

$$m\ddot{\widetilde{z}} = -mg + \frac{b_m}{z_0} - \frac{b_m}{z_0^2}\widetilde{z} + \frac{b_s}{z_0}\widetilde{i} - d\dot{\widetilde{z}}$$

Self-assessment

Can you verify that

$$-mg + \frac{b_m}{z_0} = 0,$$

and explain why it has to be as such? If so, good sign - if not, better to re-study what equilibria and ODEs are

The simplified LTI model

$$m\ddot{\tilde{z}} + d\dot{\tilde{z}} + \frac{b_m}{z_0^2}\tilde{z} = \frac{b_s}{z_0}\tilde{i}$$

Laplace-ing the LTI model

$$m\ddot{\tilde{z}} + d\dot{\tilde{z}} + \frac{b_m}{z_0^2}\tilde{z} = \frac{b_s}{z_0}\tilde{i}$$

becomes

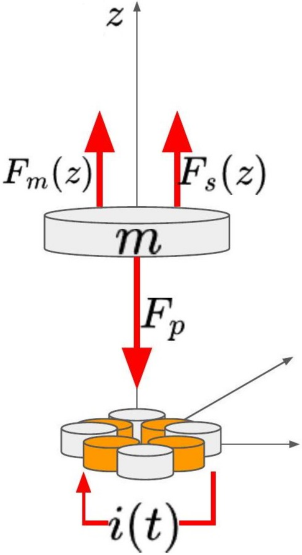
$$Z(s) = M(s) + G(s)I(s)$$

with

$$M(s) = \frac{-msz(0) - (m+d)\dot{z}(0)}{\left(ms^2 + ds + \frac{b_m}{z_0^2}\right)} \qquad G(s) = \frac{\frac{b_s}{z_0}}{\left(ms^2 + ds + \frac{b_m}{z_0^2}\right)}$$

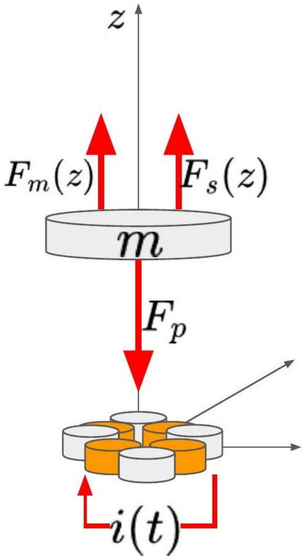
?

How do we simulate the system in Matlab?

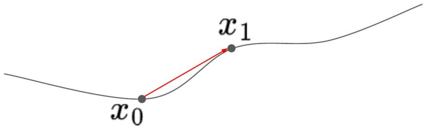


use simulink or numerically integrate the ODE

Simulation by numerically integrating the SISO ODE



$$\dot{x} = f(x, u) \implies x_1 = x_0 + \int_{t_0}^{t_1} f(x_t, u)\,dt$$



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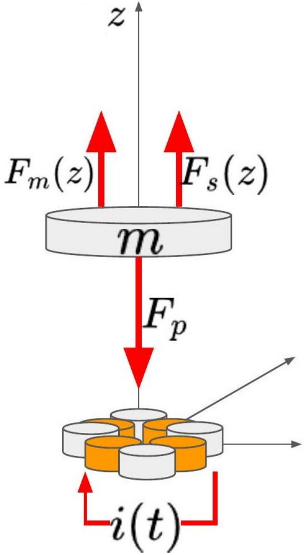
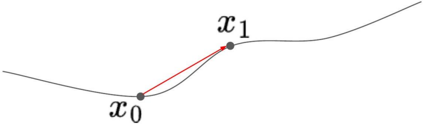


$$\dot{x} = f(x, u) \implies x_1 = x_0 + \int_{t_0}^{t_1} f(x_t, u)dt$$



Matlab will compute this integral numerically!

Writing our SISO system in a state-space form

$$x = \begin{bmatrix} z \\ \dot{z} \end{bmatrix} \implies \dot{x} = \begin{bmatrix} \dot{z} \\ \ddot{z} \end{bmatrix}$$

Writing our SISO system in a state-space form

$$x = \begin{bmatrix} z \\ \dot{z} \end{bmatrix} \implies \dot{x} = \begin{bmatrix} \dot{z} \\ \ddot{z} \end{bmatrix}$$

this means

$$\dot{x} = \begin{bmatrix} \dot{z} \\ -g + \frac{1}{m}\left(-F_p + F_m(z) + F_s(z, i) - d\dot{z}\right) \end{bmatrix}$$

Writing our SISO system in a state-space form

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the uncontrolled system is barely stable, we shall devise a feed-back control strategy that increases the basin of stability!

First assignment (max 0.2 points)

Show with some plots that the chosen equilibrium point is asymptotically stable for the nonlinear SISO model by:

- initializing the simulation with the levitating magnet above the equilibrium and with zero velocity
- simulating the system for a few seconds without the feedback control strategy
- finding for which $\widetilde{z}(0)$'s the trajectories converge to the chosen equilibrium
- reporting plots of the system trajectories for the biggest $\widetilde{z}(0)$'s for which the system converges to the wished equilibrium
- commenting the results

Second assignment (max 0.2 points)

Develop a P controller (purely feedback-based, no feed-forward actions) that increments the stability basin of the system by using a root locus design-based approach. More precisely, (try to) find a P gain for which the settling time is smaller than 1 second when the reference makes a step from the original equilibrium of amplitude 0.02 meters (i.e., when $z_{\text{ref}}(t) = z_0 + 0.02\text{step}(t)$).

Report the steps you followed for designing such a controller, the controller itself (if you can find it), and the simulation results using the nonlinear SISO model. If you are not able to find it and believe it is not possible to find such a thing, motivate why you think it is not possible to find such a thing.

Third assignment (max 0.2 points)

Develop another controller - this time a PID - using all the tools that you may find useful. Report the steps you followed for designing such a controller, the controller itself, and the simulation results using the nonlinear SISO model.

Hints:

- choose which performance indexes the wished controller should have before starting designing the controller, but consider refining these indexes after the first tests (especially if you realize you have been a bit too optimistic in what to get)
- be sure to have sufficiently large stability margins

Fourth assignment (max 0.2 points)

Choose whichever controller you prefer, and verify how high you are able to push the levitating magnet with an opportune reference (initialize the system at the equilibrium used until now).
Report plots of the reference used to push the levitating magnet high, and how high it actually went.

Fifth assignment (max 0.2 points)

Choose whichever controller you prefer, and verify how well the levitating magnet can track each of these reference trajectories:

1. $z_{\mathrm{ref}}(t) = z_0 + 0.01 \sin(2\pi t)$
2. $z_{\mathrm{ref}}(t) = z_0 + 0.01 \mathrm{squarewave}(2\pi t)$

As an indicative performance index, you will have a "good enough" controller from the commissioner point of view if the tracking error is going to be less than 2 millimeters for each time step. The smaller the average tracking error, the better the controller! Report plots of the simulated trajectories and tracking errors, and discuss the results you got.

Sixth assignment (optional, no points, and only for the brave ones)

Test your controller on the full original simulator, the one where the control input signals are four (the current in each solenoid). Note that in that simulator $i(t) \in \mathbb{R}^4$ and the state encodes the full pose of the system, i.e., $x \in \mathbb{R}^6$.

Report the choices you took to make the controller developed for the simplified SISO system be a controller for the full system, and which results you got.