Pole Placement with PID Controllers

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Main ILO of sub-module "Pole Placement with PID Controllers"

Design a PID controller to place the closed-loop poles at desired locations

Why Pole Placement with PID?

- Pole placement helps us control system response: speed, overshoot, damping.
- PID controllers can shift poles by adjusting K_P, K_I, K_D.
- This is useful when we want specific time-domain behavior (e.g., settling time, overshoot).

Worked Example: First-Order Plant

Given: $G(s) = \dfrac{1}{s+1}$ (first-order system)

Goal: Place closed-loop pole at $s = -4$

Try: Use a proportional controller: $C(s) = K_P$

Closed-loop: $\dfrac{K_P G(s)}{1 + K_P G(s)} = \dfrac{K_P}{s + 1 + K_P}$

Compare: $s + (1 + K_P) = s + 4 \Rightarrow K_P = 3$

Second-Order System and PID

Given: $G(s) = \dfrac{1}{s(s+1)}$ (2nd order, non-minimum phase)

Controller: $C(s) = K_P + \dfrac{K_I}{s} + K_D s$

Closed-loop char. poly: Denominator of $1 + C(s)G(s)$

Choose desired poles (e.g., $s = -2 \pm j2$):

Desired poly: $s^2 + 4s + 8$

Solve for K_P, K_I, K_D so that closed-loop denominator matches this.

Summarizing

From desired poles to PID parameters

- Pick desired poles based on time response specs
- Derive desired characteristic polynomial
- Write closed-loop transfer function with PID
- Match polynomials & solve for K_P, K_I, K_D
- Note that you may not be able to place the poles where you want (i.e., the system above to do not have solutions)

Most important python code for this sub-module

Python Enables Symbolic Matching of PID Coefficients

`sympy`

Self-assessment material

Question 1

What is the first step in designing a PID controller using pole placement?

Potential answers:

I: Tune K_P using trial-and-error
II: Write the plant transfer function in state-space
III: Choose desired closed-loop poles based on time-domain specs
IV: Set the integral gain to zero initially

Question 2

What is the main goal of pole placement when designing a controller?

Potential answers:

I: To cancel all poles and zeros of the system
II: To achieve desired time-domain behavior such as settling time and overshoot
III: To make the transfer function purely algebraic
IV: To eliminate the need for feedback
V: I do not know

Question 3

How does the derivative term (K_D) in a PID controller primarily affect the pole placement of a system?

Potential answers:

I: It shifts the system poles toward the imaginary axis
II: It always eliminates steady-state error
III: It has no influence on the pole placement
IV: It influences the damping and stability by modifying the characteristic equation
V: I do not know

Question 4

What is the key mathematical operation used to design PID gains through pole placement?

Potential answers:

I: Taking the inverse Laplace transform of the plant
II: Eliminating zeros from the open-loop transfer function
III: Matching the closed-loop characteristic polynomial to a desired one
IV: Factorizing the numerator of the open-loop transfer function
V: I do not know

Question 5

In a first-order system controlled by a proportional gain K_P, what is the effect of increasing K_P?

Potential answers:

I: The pole moves further left on the real axis, increasing system speed
II: The pole becomes complex and causes oscillations
III: The system gain decreases and response slows down
IV: The zero of the system moves into the right-half plane
V: I do not know

Question 6

Which of the following best describes the correct order of steps for PID pole placement design?

Potential answers:

I: Compute the system output first, then choose PID gains, then set desired poles

II: Start with experimental PID gains, simulate, and refine based on intuition

III: Choose desired poles, derive the corresponding characteristic polynomial, and match it with the actual closed-loop polynomial to solve for gains

IV: Eliminate the need for poles by transforming to frequency domain

V: I do not know

Recap of sub-module "Pole Placement with PID Controllers"

- Pole placement allows us to achieve desired dynamics
- PID gains shift the closed-loop poles
- Match desired characteristic polynomial with actual one
- Use symbolic or numerical tools to solve for K_P, K_I, K_D

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