Cosa implica fare delle approssimazioni ai poli dominanti

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risposta al gradino	u1, e1
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Main ILO of sub-module
“Cosa implica fare delle approssimazioni ai poli dominanti”

Discuss how the step response of a system may change when applying dominant pole approximation to a transfer function with multiple poles and zeros

Roadmap

- cosa succede se ignoriamo un polo?
- cosa succede se ignoriamo un polo ed uno zero?

Cosa succede se l'approssimazione ai poli dominanti non e' valida?

questo modulo = cosa succede in alcuni casi specifici

Caso 1: ci sono due poli reali distinti

FdT originale:

$$W(s) = \frac{p}{(s+1)(s+p)} = \frac{1}{(1+s)(1+s\tau)} \qquad \tau = 1/p > 0$$

FdT approssimata:

$$\widehat{W}(s) = \frac{1}{s+1}$$

Che errore si introduce ad usare la FdT approssimata invece dell'originale?

caso risposta al gradino

$$Y_f(s) = \frac{p}{(s+1)(s+p)}\frac{1}{s} = \frac{1}{s} - \frac{\frac{p}{p-1}}{s+1} + \frac{\frac{1}{p-1}}{s+p}$$

implica

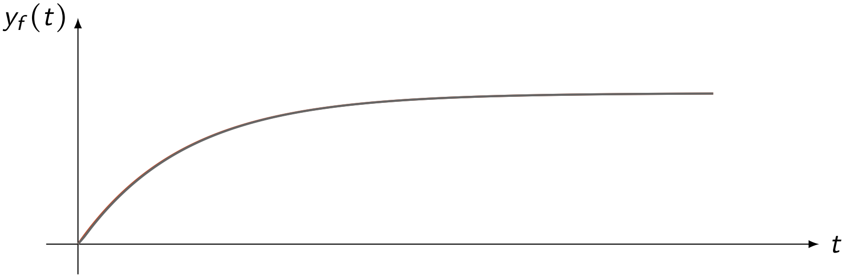
$$y_f(t) = 1 - \frac{p}{p-1}e^{-t} + \frac{1}{p-1}e^{-pt} \qquad t \geq 0$$

mentre con il solo polo dominante la risposta sarebbe stata

$$\frac{1}{s+1}\frac{1}{s} \quad \mapsto \quad 1 - e^{-t}$$

Che errore si introduce ad usare la FdT approssimata invece dell'originale?

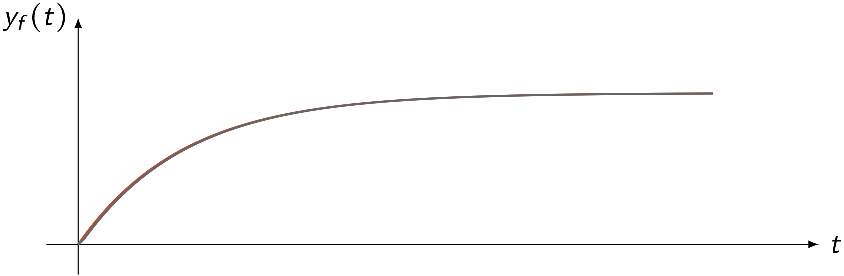
caso risposta al gradino



$$\frac{1}{s+1}\frac{1}{s} \qquad \frac{100}{(s+1)(s+100)}\frac{1}{s}$$

Che errore si introduce ad usare la FdT approssimata invece dell'originale?

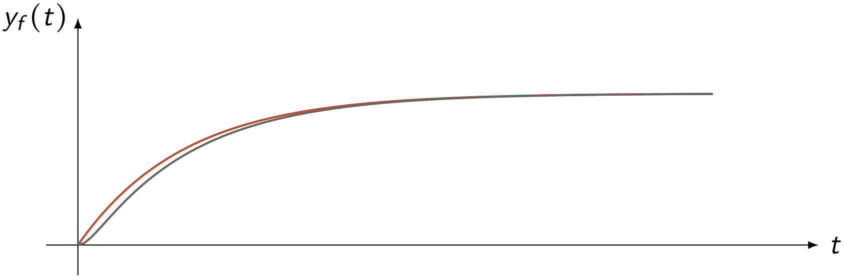
caso risposta al gradino



$$\frac{1}{s+1}\frac{1}{s} \qquad \frac{50}{(s+1)(s+50)}\frac{1}{s}$$

Che errore si introduce ad usare la FdT approssimata invece dell'originale?

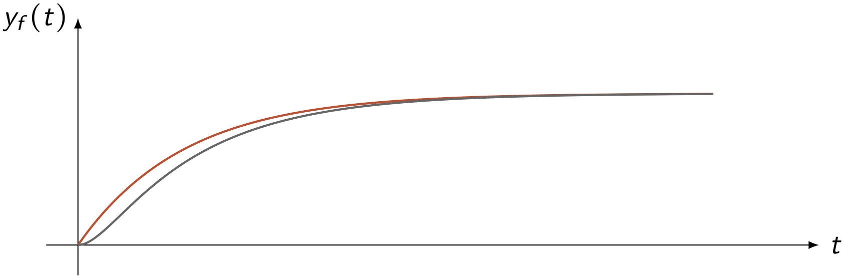
caso risposta al gradino



$$\frac{1}{s+1}\frac{1}{s} \qquad \frac{10}{(s+1)(s+10)}\frac{1}{s}$$

Che errore si introduce ad usare la FdT approssimata invece dell'originale?

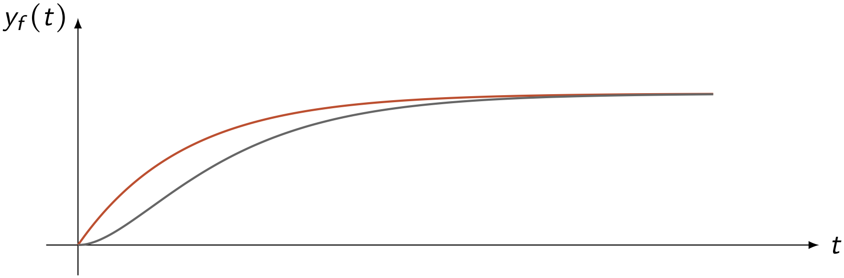
caso risposta al gradino



$$\frac{1}{s+1}\frac{1}{s} \qquad \frac{5}{(s+1)(s+5)}\frac{1}{s}$$

Che errore si introduce ad usare la FdT approssimata invece dell'originale?

caso risposta al gradino



$$\frac{1}{s+1}\frac{1}{s} \qquad \frac{2}{(s+1)(s+2)}\frac{1}{s}$$

Summarizing, ignorare il polo non dominante significa considerare una risposta anticipata rispetto a quella che si avrebbe

$$\frac{1}{s+1}\frac{1}{s} \qquad \frac{p}{(s+1)(s+p)}\frac{1}{s}$$

$$1 - e^{-t} \qquad 1 - \frac{p}{p-1}e^{-t} + \frac{1}{p-1}e^{-pt}$$

?

E se c'e' anche uno zero, ed ignoriamo anche quello?

$$\frac{1}{s+1} \cdot \frac{1}{s} \qquad\qquad \frac{p}{z}\frac{s+z}{(s+1)(s+p)} \cdot \frac{1}{s}$$

E se c'e' anche uno zero, ed ignoriamo anche quello?

$$\frac{1}{s+1} \cdot \frac{1}{s} \qquad \frac{p}{z} \frac{s+z}{(s+1)(s+p)} \cdot \frac{1}{s}$$

$$1 - e^{-t} \qquad 1 - \frac{p}{p-1} \frac{z-1}{z} e^{-t} + \frac{1}{p-1} \frac{z-p}{z} e^{-pt}$$

i.e., si modificano i residui

Summarizing

Discuss how the step response of a system may change when applying dominant pole approximation to a transfer function with multiple poles and zeros

Self-assessment material

Question 1

What is a necessary condition for the dominant pole approximation to be valid?

Potential answers:

I: The dominant pole must be closer to the origin than any zero.
II: The non-dominant poles must be significantly faster than the dominant pole.
III: The dominant pole must be complex with a small imaginary part.
IV: The gain of the system must be normalized to 1.
V: I do not know

Question 2

Which effect is typically observed when a non-dominant pole is ignored in a second-order system during a step response analysis?

Potential answers:

I: The system becomes unstable.
II: The system appears to respond faster than it actually does.
III: The steady-state value is significantly altered.
IV: The response becomes oscillatory even if the original system is overdamped.
V: I do not know

Question 3

What is the primary modeling error introduced by using a dominant pole approximation for the transfer function $\frac{p}{(s+1)(s+p)}$?

Potential answers:

I: The final value of the response is incorrect.

II: The early-time dynamics (initial transient) are not accurately captured.

III: The approximation leads to an unstable system.

IV: The system gain becomes infinite.

V: I do not know

Question 4

In the transfer function $\frac{p}{(s+1)(s+p)}$, what happens to the response as $p \to \infty$?

Potential answers:

I: The response converges to that of the system with transfer function $\frac{1}{s+1}$.

II: The response diverges and becomes unstable.

III: The system behaves like a pure integrator.

IV: The steady-state value drops to zero.

V: I do not know

Question 5

In the Laplace domain, why does the approximation $\frac{p}{(s+1)(s+p)} \approx \frac{1}{s+1}$ become worse for small p?

Potential answers:

I: Because both poles move closer together and cancel each other out.
II: Because the gain of the system changes drastically.
III: Because the time constant of the neglected pole becomes comparable to the dominant one.
IV: Because the numerator introduces a zero near the origin.
V: I do not know

Recap of the module
“Cosa implica fare delle approssimazioni ai poli dominanti”

- ignorare un polo significa tipicamente (manon sempre) considerare una risposta piu’ veloce di quella che si avrebbe originariamente
- ignorare uno zero significa fare un ulteriore errore

?