Analisi del transitorio per i sistemi del primo ordine

Contents map

developed content units	taxonomy levels
costante di tempo	u1, e1

prerequisite content units	taxonomy levels
risposta forzata	u1, e1
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Main ILO of sub-module "Analisi del transitorio per i sistemi del primo ordine"

- Model first-order LTI systems using their dominant pole approximation
- Derive the step response equation for canonical first-order systems
- Calculate the time constant from system parameters and vice versa
- Interpret first-order system behavior through time-domain plots

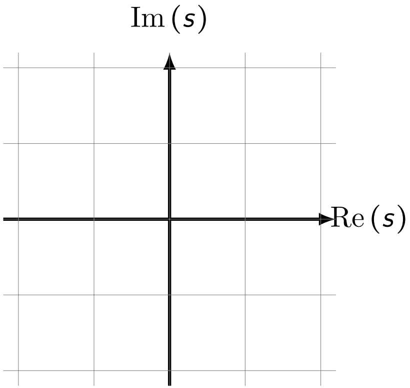
Roadmap

- conti
- grafici

Caso piu' semplice: polo dominante = singolo e reale

$\Longrightarrow$ approssimazione di $W(s) = \widehat{W}(s) = \dfrac{K}{s-p}$

con $W(0) = \widehat{W}(0)$



Risposta (forzata) al gradino per i sistemi del primo ordine - conti

$$W(s) = \frac{K}{s-p} = \frac{K}{-p(1-s/p)} = \frac{K_B}{1+\tau s}$$

$$K_B = -\frac{K}{p} \quad \tau = -\frac{1}{p} > 0$$

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Se $u(t) = \delta^{(-1)}(t)$, allora $U(s) = \frac{1}{s}$ e quindi

$$Y_f(s) = W(s)\frac{1}{s} = \frac{A}{s} + \frac{B}{s+1/\tau}$$

con $A = W(0) = K_B$ e $B = (s+1/\tau)Y_f(s)\big|_{s=-1/\tau} = -K_B$.

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con $A = W(0) = K_B$ e $B = (s+1/\tau)Y_f(s)|_{s=-1/\tau} = -K_B$. Quindi

$$y_f(t) = K_B\left(1 - e^{-\frac{t}{\tau}}\right) \quad t \geq 0$$

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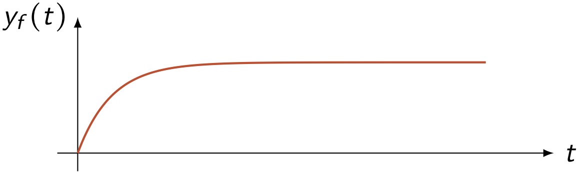

Relazioni ed approssimazioni da ricordare per questo caso

$$y_f(t) = K_B\left(1 - \exp\left(-\frac{t}{\tau}\right)\right) \quad t \geq 0$$

implica

$$S = 0$$

$$U = 0$$

$$T_s = \tau \ln 9 \simeq 2.2\tau = -\frac{2.2}{p}$$

$$T_a = \tau \ln 20 \simeq 3\tau = -\frac{3}{p}$$

Summarizing

Model first-order LTI systems using their dominant pole approximation

Derive the step response equation for canonical first-order systems

Calculate the time constant from system parameters and vice versa

Interpret first-order system behavior through time-domain plots

Most important python code for this sub-module

Control systems library

- `control.step_response()`
- `control.step_info()`

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Self-assessment material

Question 1

For a first-order system with transfer function $W(s) = \dfrac{K}{s-p}$, which of the following expressions correctly represents the step response?

Potential answers:

I: $y_f(t) = K_B \cdot e^{-\frac{t}{\tau}}$ for $t \geq 0$
II: $y_f(t) = K_B \cdot \left(1 - e^{-t\cdot\tau}\right)$ for $t \geq 0$
III: $y_f(t) = K_B \cdot \left(1 - e^{-\frac{t}{\tau}}\right)$ for $t \geq 0$
IV: $y_f(t) = K_B \cdot \left(e^{-\frac{t}{\tau}} - 1\right)$ for $t \geq 0$
V: I do not know

Question 2

The settling time T_a for a first-order system with time constant τ and pole at p is approximately:

Potential answers:

I: $T_a \approx \tau$
II: $T_a \approx 2\tau$
III: $T_a \approx 3\tau$
IV: $T_a \approx 5\tau$
V: I do not know

Question 3

What is the mathematical expression for the rise time T_s of a first-order system in terms of its time constant τ?

Potential answers:

I: $T_s = \tau$
II: $T_s = \tau \ln 9 \approx 2.2\tau$
III: $T_s = \tau \ln 20 \approx 3\tau$
IV: $T_s = \tau \ln 4 \approx 1.4\tau$
V: I do not know

Question 4

What is the overshoot value S for a first-order system with a single real pole?

Potential answers:

I: $S = 0$

II: $S = 0.05$

III: $S = e^{-\pi} \approx 0.043$

IV: $S = \frac{1}{\tau}$

V: I do not know

Question 5

Given a first-order system with a stable pole at $p = -2$, what is the value of its time constant τ?

Potential answers:

I: $\tau = -2$
II: $\tau = 2$
III: $\tau = 0.5$
IV: $\tau = -0.5$
V: I do not know

Recap of module “Analisi del transitorio per i sistemi del primo ordine”

- La risposta al gradino di un sistema del primo ordine ha una forma esponenziale crescente verso un valore asintotico.
- Il sistema non presenta né sovraelongazione né sottoelongazione.
- Tutti i parametri del transitorio possono essere espressi in funzione della costante di tempo τ.
- La costante di tempo può essere ricavata anche graficamente, e dà un'indicazione chiara della "prontezza" del sistema.

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