when is linearizing meaningful

Contents map

developed content units	taxonomy levels
linearization	u1, e1

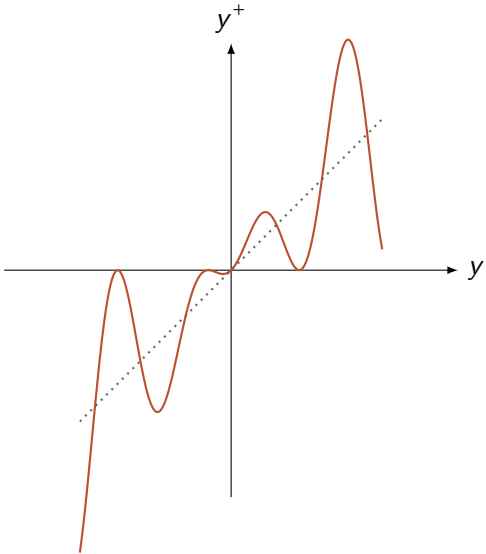
prerequisite content units	taxonomy levels
RR	u1, e1

Main ILO of sub-module "when is linearizing meaningful"

Assess the validity of the approximation introduced when linearizing a nonlinear RR around an equilibrium point

Evaluate the meaning and applicability of linearization in different contexts, discussing when it provides a reasonable approximation and when it does not

Discussion: around which equilibria may we consider linearizations “good”?



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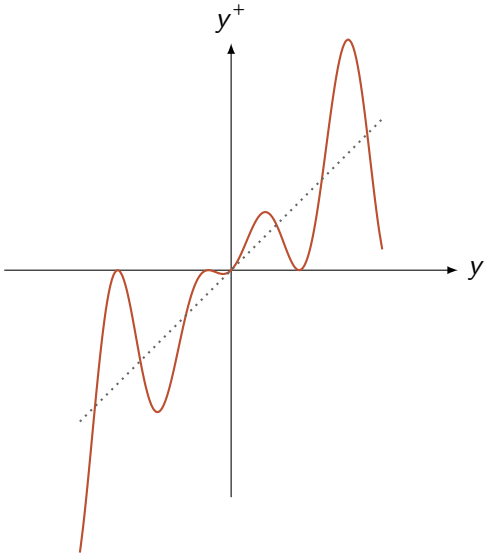
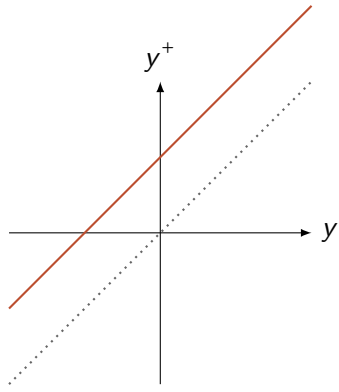


also for the ‘unstable’ equilibria the approximation may be a good one - depends on the time horizon under consideration and how close y_0 is to the equilibrium

Discussion: is it always meaningful to linearize?



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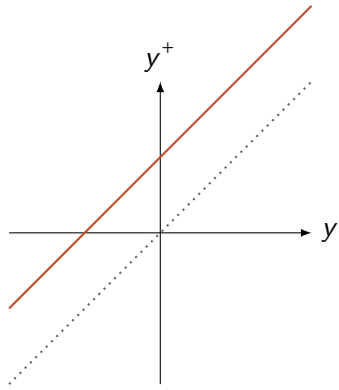
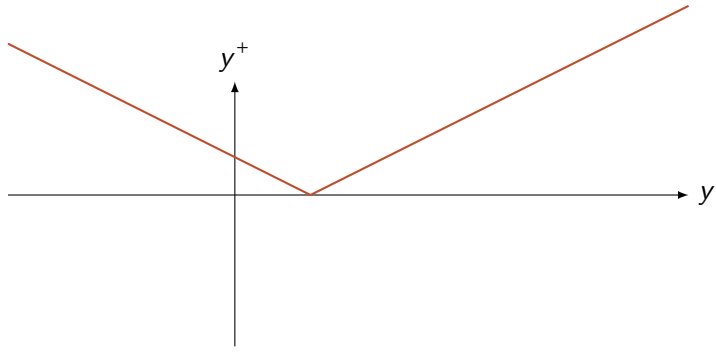


in this case we do not have equilibria

Discussion: and here, can we linearize?



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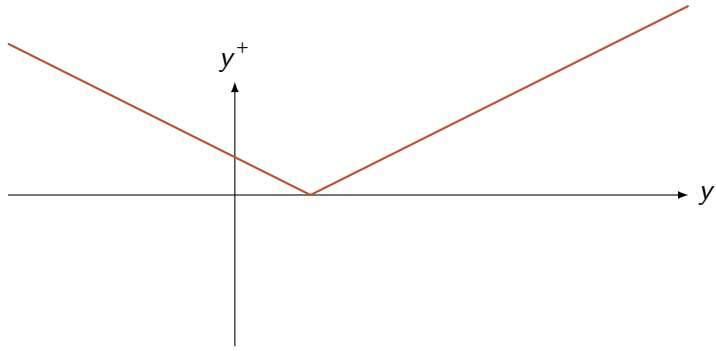
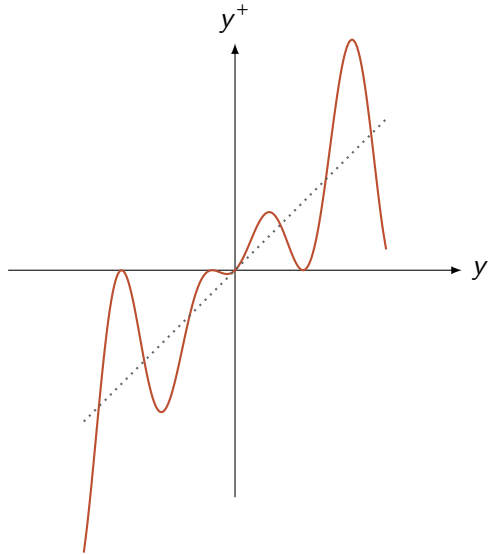


in this case we cannot compute the first derivative

Discussion: can we trust the stable linearized system for this case?



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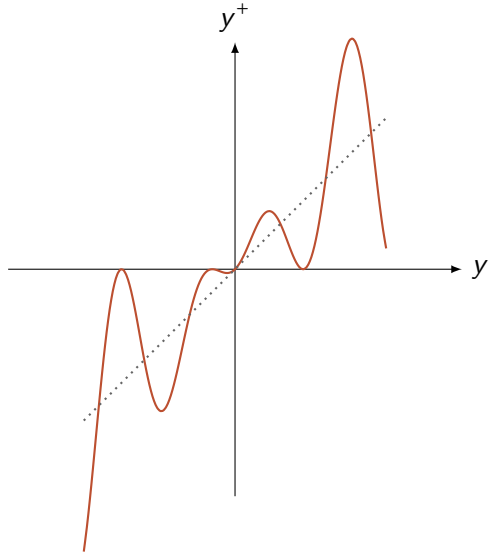

Summarizing

Assess the validity of the approximation introduced when linearizing a nonlinear RR around an equilibrium point

Evaluate the meaning and applicability of linearization in different contexts, discussing when it provides a reasonable approximation and when it does not

- if we have an asymptotically stable equilibrium, the approximation improves in time
- if we have an unstable equilibrium, the approximation degrades in time
- the closer we start from the equilibrium, the better
- the bigger the curvature of the RR, the more "local" the results will be

Most important python code for this sub-module

This will do everything for you

`https://python-control.readthedocs.io/en/latest/generated/control.linearize.html`

though it is dangerous to use tools without knowing how they work

Self-assessment material

Question 1

Which of the following statements about linearization around an equilibrium point is correct?

Potential answers:

I: Linearization provides a good approximation for any nonlinear system at any point.

II: Linearization is only useful for stable equilibria and does not work for unstable ones.

III: Linearization can be a good approximation near both stable and unstable equilibria, depending on the time horizon and initial conditions.

IV: Linearization is only valid if the system has no nonlinear terms.

V: I do not know

Question 2

Under which condition is linearization not possible?

Potential answers:

I: If the equilibrium is unstable.
II: If the equilibrium is stable but far from the origin.
III: If the system's function is not differentiable at the equilibrium point.
IV: If the system is highly nonlinear.
V: I do not know

Question 3

How does the curvature of the nonlinear system affect the validity of linearization?

Potential answers:

I: Curvature does not affect the validity of linearization.
II: The larger the curvature, the more accurate the linearized model.
III: The larger the curvature, the more local the validity of the linearized model.
IV: Linearization is only valid when curvature is zero.
V: I do not know

Question 4

What does the size of the basin of attraction tell us about the linearized model?

Potential answers:

I: A small basin of attraction means the linearized model is only valid in a very restricted region.

II: A small basin of attraction means the system is globally stable.

III: A large basin of attraction makes linearization unnecessary.

IV: The basin of attraction does not affect the validity of the linearization.

V: I do not know

Question 5

In which of the following cases is linearization not meaningful?

Potential answers:

I: When the system has no equilibrium points.
II: When the equilibrium point is unstable.
III: When the system is nonlinear.
IV: When the system has high curvature.
V: I do not know

Recap of sub-module "when is linearizing meaningful"

- be careful when using a linearized system - be always aware of where it comes from

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