explain what BIBO stability means

Contents map

developed content units	taxonomy levels
unbounded signal	u1, e1
bounded signal	u1, e1
BIBO stability	u1, e1

prerequisite content units	taxonomy levels
ODE	u1, e1

Main ILO of sub-module "explain what BIBO stability means"

Graphically explain the definition of BIBO (Bounded-Input Bounded-Output) stability and its connection to system behavior

Give examples of systems that are BIBO stable or not, and motivate their properties with physical intuitions

Definitions: bounded and unbounded signals



bounded: I can find an M for which |y(t)| < M for every t. **Unbounded** otherwise

Intuition



simplified model = single integrator: $\dot{v} = bu$

Intuition



simplified model = single integrator: $\dot{v} = bu$

$$u(t) = \text{const.} \neq 0 \implies \lim_{t \to +\infty} v(t) = ?$$

Interesting difference

 $\dot{v} = 0v + bu \implies \text{constant non null } u(t) \text{ causes } v(t) \text{ unbounded}$

 $\dot{v} = av + bu, a < 0 \implies \text{constant non null } u(t) \text{ causes } v(t) \text{ bounded}$

Visualizing this with avocados

for $\dot{v} = 0v + bu$ some bounded u(t)'s lead to unbounded v(t)'s



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Visualizing this with avocados

for $\dot{v} = 0v + bu$ not all the bounded u(t)'s lead to unbounded v(t)'s, though



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Visualizing this with avocados

for $\dot{v} = av + bu$, a < 0 all the bounded u(t)'s lead to bounded v(t)'s



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Important difference: the two systems have two different "stability" properties relative to how inputs get mapped into outputs!

Remark

- "Stability" referring to specific equilibria:
 - simply stable equilibrium
 - convergent equilibrium
 - asymptotically stable equilibrium
- "Stability" referring to specific systems:
 - Bounded Input Bounded Output (BIBO) stable systems (we are seeing this now)
 - Input to State Stable (ISS) systems (we will not see this in this module)

Definition: BIBO stability

If $\dot{y} = f(y, u)$ is so that if u is bounded then y will be for sure bounded, then that system is said BIBO stable



Definition: BIBO stability

If $\dot{y} = f(y, u)$ is so that there exists at least one u bounded for which the corresponding y will be unbounded, then that system is said **not** BIBO stable



Is $\dot{y} = y + u$ BIBO stable?

Is $\dot{y} = yu$ BIBO stable?

Is $\dot{y} = u$ BIBO stable?

Is $\dot{y} = -y$ BIBO stable?

Summarizing

Graphically explain the definition of BIBO (Bounded-Input Bounded-Output) stability and its connection to system behavior

Give examples of systems that are BIBO stable or not, and motivate their properties with physical intuitions

- if starting from the kernel of the input-avocado we are guaranteed to end up in the kernel of the output-avocado, then we have BIBO stability
- if not, not
- BIBO stability is a concept that relates to non-autonomous system and it is not necessary that it it LTI, may also be nonlinear. But it needs to have at least one input

Most important python code for this sub-module

No dedicated python libraries for this

... but one can use the control library for checking the properties of the transfer function or impulse response of the system, if LTI of course

Self-assessment material

Which of the following statements best describes BIBO (Bounded-Input Bounded-Output) stability?

- I: A system is BIBO stable if all outputs remain constant regardless of the input.
- II: A system is BIBO stable if every bounded input leads to a bounded output.
- III: A system is BIBO stable if it has at least one bounded output for an unbounded input.
- IV: A system is BIBO stable if it is asymptotically stable.
- $V{:} \ I \ do \ not \ know$

Which of the following systems is NOT BIBO stable?

I:
$$\dot{y} = -2y + u$$
.
II: $\dot{y} = y + u$.
III: $\dot{y} = -y + 3u$.
IV: $\dot{y} = -0.5y + u$.
V: I do not know

Which graphical interpretation correctly illustrates a system that is NOT BIBO stable?

- I: A system where all bounded inputs correspond to bounded outputs.
- II: A system where at least one bounded input results in an unbounded output.
- III: A system where all unbounded inputs lead to unbounded outputs.
- IV: A system where the impulse response is always decreasing over time.
- $V{:}\ I\ do\ not\ know$

A spacecraft is modeled as a single integrator $\dot{v} = bu$. What can be said about its BIBO stability?

- I: The system is NOT BIBO stable because a constant nonzero input leads to an unbounded velocity.
- II: The system is BIBO stable because velocity is a smooth function of time.
- III: The system is BIBO stable because acceleration remains bounded.
- IV: The system is BIBO stable because it eventually reaches a steady-state velocity.
- V: I do not know

Can one define the BIBO properties of an autonomous system?

- I: No, BIBO stability is defined in terms of input-output behavior.
- II: Yes, every system has a well-defined BIBO stability property.
- III: Only if the system is linear and time-invariant.
- IV: Yes, but only for discrete-time systems.
- V: I do not know

If a system has two inputs, can it be BIBO stable with respect to one input but not the other?

- I: Yes, different inputs can excite different dynamics in the system.
- II: No, BIBO stability is a system-wide property.
- III: Only if the system is nonlinear.
- IV: Only for discrete-time systems.
- V: I do not know

BIBO stability properties are connected to equilibrium points, so if a system has three equilibria, do we need to analyze BIBO stability separately for each equilibrium?

- I: No, BIBO stability is an input-output property and does not depend on equilibria.
- II: Yes, because each equilibrium defines a different stability region.
- III: Only if the system is nonlinear.
- IV: Only for continuous-time systems.
- V: I do not know

Is an integrator a BIBO stable system?

- I: No, an integrator accumulates input over time, leading to unbounded output for bounded input.
- II: Yes, because the output remains predictable.
- III: Only in discrete-time systems.
- IV: Only if the initial condition is zero.
- V: I do not know

Recap of sub-module "explain what BIBO stability means"

- BIBO stability means "a bounded input must imply a bounded output"
- it is a concept that in general it is disconnected to that of marginal stability / convergence of an equilibrium

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