Systems Laboratory, Spring 2025

Damiano Varagnolo – CC-BY-4.0

when is linearizing meaningful

Contents map

developed content units	taxonomy levels
linearization	u1, e1

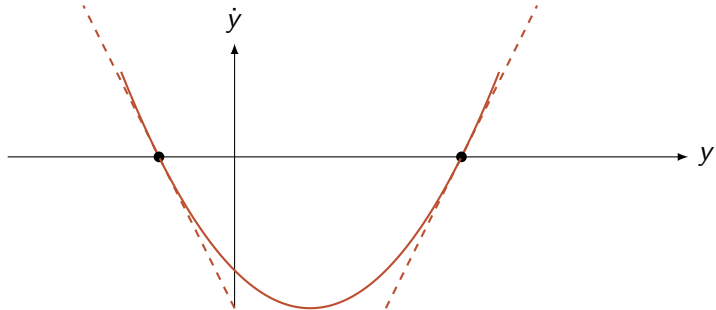
prerequisite content units	taxonomy levels
ODE	u1, e1

Main ILO of sub-module "when is linearizing meaningful"

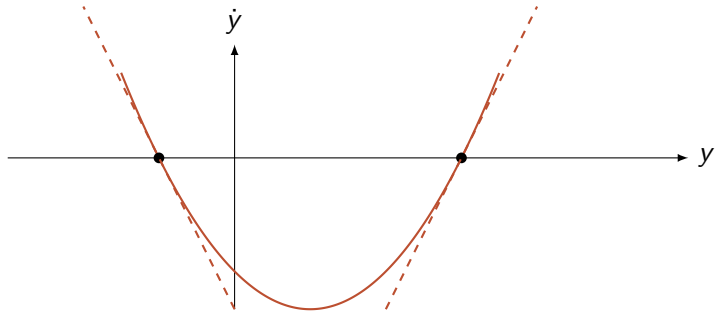
Assess the validity of the approximation introduced when linearizing a nonlinear ODE around an equilibrium point

Evaluate the meaning and applicability of linearization in different contexts, discussing when it provides a reasonable approximation and when it does not

Discussion: around which equilibrium may we consider this model approximation a “good one”?

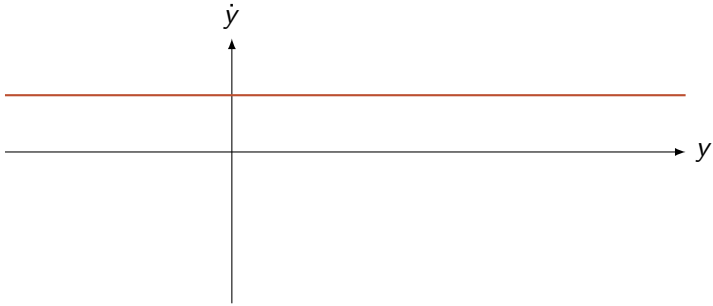


Discussion: around which equilibrium may we consider this model approximation a “good one”?

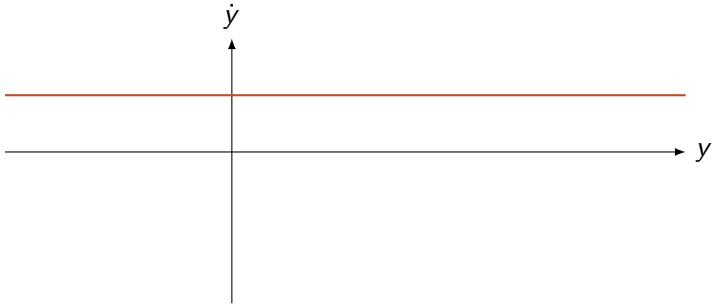


also for the ‘unstable’ equilibrium the approximation may be a good one - depends on the time horizon under consideration and how close y_0 is to the equilibrium

Discussion: is it always meaningful to linearize?

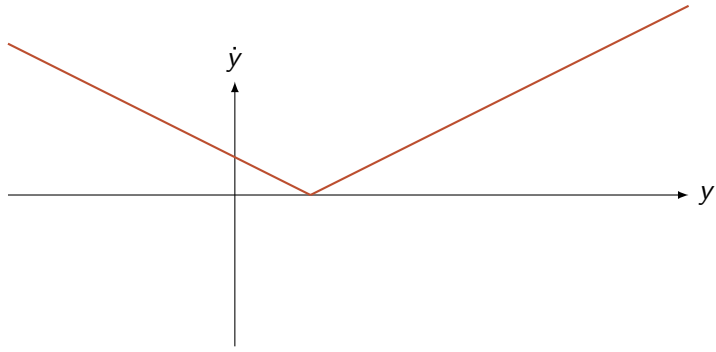


Discussion: is it always meaningful to linearize?

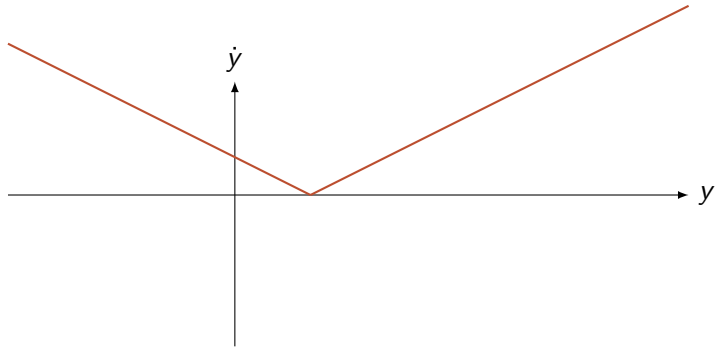


in this case we do not have equilibria

Discussion: and here, can we linearize?

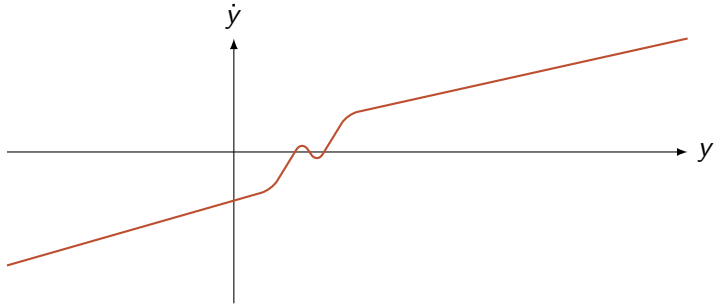


Discussion: and here, can we linearize?

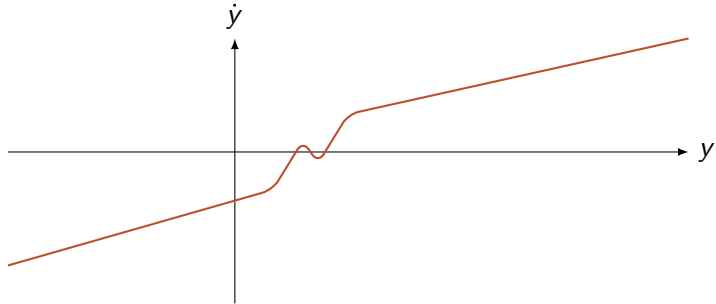


in this case we cannot compute the first derivative

Discussion: can we trust the stable linearized system for this case?



Discussion: can we trust the stable linearized system for this case?



in this case the basin of attraction is very small

Summarizing

Assess the validity of the approximation introduced when linearizing a nonlinear ODE around an equilibrium point

Evaluate the meaning and applicability of linearization in different contexts, discussing when it provides a reasonable approximation and when it does not

- if we have an asymptotically stable equilibrium, the approximation improves in time
- if we have an unstable equilibrium, the approximation degrades in time
- the closer we start from the equilibrium, the better
- the bigger the curvature of the ODE, the more "local" the results will be

Most important python code for this sub-module

This will do everything for you

`https://python-control.readthedocs.io/en/latest/generated/control.linearize.html`

though it is dangerous to use tools without knowing how they work

Self-assessment material

Question 1

When linearizing a nonlinear ODE around an equilibrium point, which of the following conditions ensures that the approximation improves over time?

Potential answers:

I: The equilibrium point is unstable.
II: The equilibrium point is asymptotically stable.
III: The ODE has a high curvature near the equilibrium point.
IV: The initial point is far from the equilibrium.
V: I do not know.

Question 2

In which of the following cases is it NOT meaningful to linearize a nonlinear ODE?

Potential answers:

I: The ODE has multiple equilibrium points.
II: The ODE does not have any equilibrium points.
III: The ODE has a small basin of attraction.
IV: The ODE is highly nonlinear.
V: I do not know.

Question 3

Which of the following factors limits the validity of a linearized ODE approximation?

Potential answers:

I: The linearized system has a stable equilibrium.
II: The basin of attraction of the equilibrium is very small.
III: The ODE is continuous and differentiable.
IV: The initial point is close to the equilibrium.
V: I do not know.

Question 4

What happens to the accuracy of a linearized ODE approximation near an unstable equilibrium point over time?

Potential answers:

I: The approximation degrades over time.
II: The approximation improves over time.
III: The accuracy remains constant.
IV: The accuracy depends on the curvature of the ODE.
V: I do not know.

Question 5

Which of the following statements about linearization is true?

Potential answers:

I: Linearization is always a good approximation for any nonlinear ODE.

II: Linearization provides a better approximation when the initial point is closer to the equilibrium.

III: Linearization is only valid for ODEs with high curvature.

IV: Linearization cannot be applied to stable systems.

V: I do not know.

Recap of sub-module "when is linearizing meaningful"

- be careful when using a linearized system - be always aware of where it comes from

?