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notes

- this is the table of contents of this document; each section corresponds to a specific part of the course

what is an impulse response

notes

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notes

Main ILO of sub-module "what is an impulse response"

Describe what the impulse response of an LTI system is in practice

notes

- by the end of this module you shall be able to do this

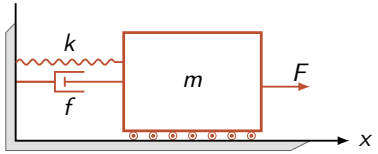
Impulse response ↔ superposition principle ↔ LTI system

talking about the impulse response of a nonlinear system is such a big mistake that may make you fail the exam on the spot

notes

- not joking here; we will see in this module that the concept is a direct consequence, and not having understood that an impulse response is meaningful only for LTI systems indicates a complete misunderstanding of the basis of the course

Practical example: spring-mass system

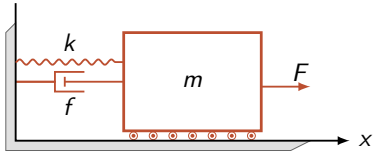


- output = position
- input = force (in Newtons)

notes

- In the mass-spring-damper system the impulse is a quick push (force times time) applied to the mass. The impulse response is the displacement of the mass over time after the push. The units of the impulse response are m/Ns.

Discretizing the spring-mass system with a Euler backwards scheme



Newton $\mapsto \ddot{y}(t) + a_1\dot{y}(t) + a_0 y(t) = b_0 u(t)$

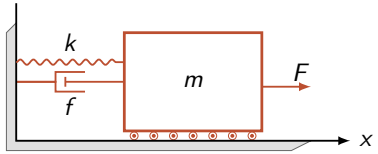
becomes, with Euler,

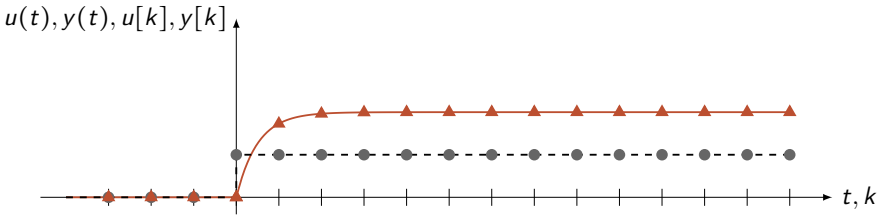
$$y^{++} + \alpha_1 y^{+} + \alpha_0 y = \beta u$$

notes

- here we are using Euler's backward method only on the only derivative we see
- arriving at the final numbers is actually just a matter of computations

What does it mean to apply a control signal $u[k] = \text{step}$?



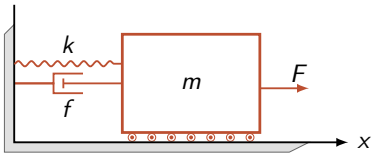


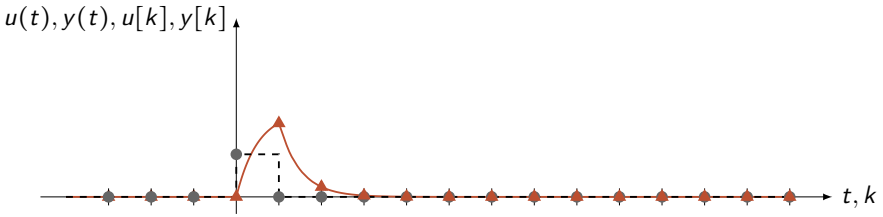
notes

- note that here the position is x, but also $y = x$

And $u[k] = 1$ only for $k = 0$, and 0 otherwise?

(i.e., $u[k] = \delta[k]$)

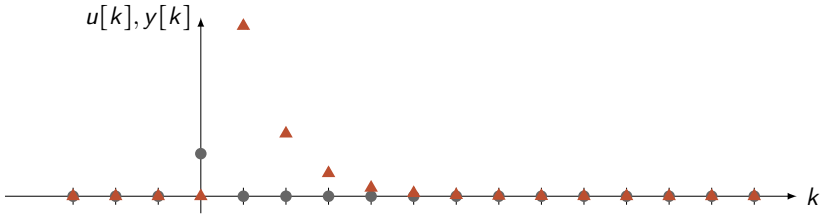




notes

- note that here the position is x, but also $y = x$
- let's moreover give a name to this signal, let's call it discrete Dirac's delta

The impulse response for DT LTI systems

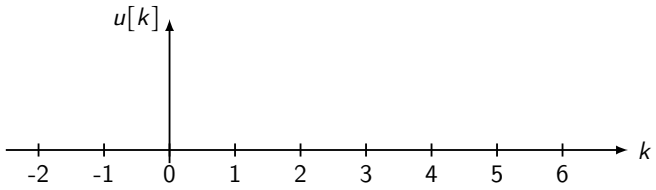


notes

- of course different systems will have different impulse responses, depending on how the system is. This one is just an example

Discussion

Where would you draw $\delta[k-4]$?



notes

- for this the spike is in $t = 10$ (because the spike is where the argument is zero, and in this case the argument is $t - 10$)

Summarizing

Describe what the impulse response of an LTI system is in practice

- an opportune behavior of the response of a LTI system to a discrete Dirac's delta

notes

- you should now be able to do this, following the pseudo-algorithm in the itemized list

Most important python code for this sub-module

notes

Important libraries / methods

- `https://docs.scipy.org/doc/scipy/reference/generated/scipy.signal.impulse.html`
- `https://python-control.readthedocs.io/en/latest/generated/control.impulse_response.html`

notes

- with these methods you can compute the impulse response of a LTI system

Self-assessment material

notes

Question 1

What is the impulse response of an LTI system?

Potential answers:

I: **(correct)** The output of the system when the input is a discrete Dirac's delta.
II: **(wrong)** The output of the system when the input is a step function.
III: **(wrong)** The output of the system when the input is a sinusoidal signal.
IV: **(wrong)** The output of the system when the input is a random signal.
V: **(wrong)** I do not know

Solution 1:

The impulse response of an LTI system is defined as the output of the system when the input is a discrete Dirac's delta ($\delta[k]$). This response characterizes the system's behavior and is fundamental to understanding LTI systems.

notes

- see the associated solution(s), if compiled with that ones :)

Question 2

Why is the impulse response meaningful only for LTI systems?

Potential answers:

I: **(wrong)** Because nonlinear systems do not have outputs.
II: **(correct)** Because the impulse response relies on the superposition principle, which is valid only for LTI systems.
III: **(wrong)** Because the impulse response is only defined for continuous-time systems.
IV: **(wrong)** Because the impulse response is too complex for nonlinear systems.
V: **(wrong)** I do not know

Solution 1:

The impulse response is meaningful only for LTI systems because it relies on the superposition principle, which holds true only for linear and time-invariant systems. Nonlinear systems do not satisfy the superposition principle, making the concept of impulse response invalid for them.

notes

- see the associated solution(s), if compiled with that ones :)

Question 3

What happens to the impulse response of a discrete-time LTI system if the input is $\delta[k-4]$?

Potential answers:

I: **(wrong)** The impulse response becomes zero.
II: **(correct)** The impulse response is shifted by 4 time units.
III: **(wrong)** The impulse response is scaled by a factor of 4.
IV: **(wrong)** The impulse response becomes nonlinear.
V: **(wrong)** I do not know

Solution 1:

For a discrete-time LTI system, if the input is $\delta[k-4]$, the impulse response is shifted by 4 time units. This is a consequence of the time-invariance property of LTI systems.

notes

- see the associated solution(s), if compiled with that ones :)

Question 4

What is the practical significance of the impulse response in analyzing LTI systems?

Potential answers:

I: **(wrong)** It allows us to ignore the system's initial conditions.
II: **(correct)** It characterizes the system's behavior and can be used to determine the output for any input.
III: **(wrong)** It ensures the system response is always sinusoidal.
IV: **(wrong)** It makes the system response independent of the input.
V: **(wrong)** I do not know

Solution 1:

The impulse response is significant because it characterizes the behavior of an LTI system. Once the impulse response is known, the output of the system for any input can be determined using convolution.

notes

- see the associated solution(s), if compiled with that ones :)

Question 5

In the context of a spring-mass-damper system, what does the impulse response represent?

Potential answers:

I: **(wrong)** The steady-state position of the mass.
II: **(correct)** The displacement of the mass over time after an instantaneous force is applied.
III: **(wrong)** The force required to keep the mass at rest.
IV: **(wrong)** The frequency of oscillation of the mass.
V: **(wrong)** I do not know

Solution 1:

In a spring-mass-damper system, the impulse response represents the displacement of the mass over time after an instantaneous force (impulse) is applied. This response captures the system's dynamic behavior.

notes

- see the associated solution(s), if compiled with that ones :)

Recap of sub-module "what is an impulse response"

- impulse responses are directly connected to step responses
- actually this connection is valid only if the system is LTI

notes

- the most important remarks from this sub-module are these ones