Table of Contents I

notes

- this is the table of contents of this document; each section corresponds to a specific part of the course

when is linearizing meaningful

notes

Contents map

developed content units	**taxonomy levels**
linearization	u1, e1

prerequisite content units	**taxonomy levels**
RR	u1, e1

notes

Main ILO of sub-module "when is linearizing meaningful"

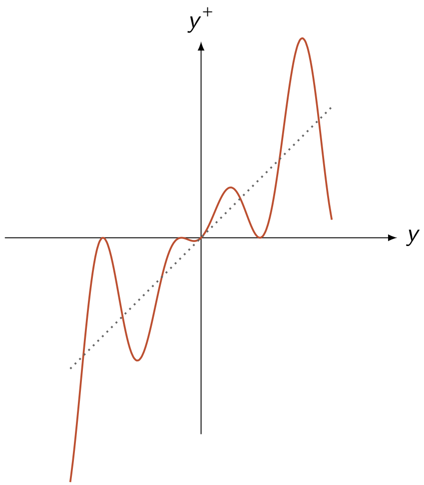
Assess the validity of the approximation introduced when linearizing a nonlinear RR around an equilibrium point

Evaluate the meaning and applicability of linearization in different contexts, discussing when it provides a reasonable approximation and when it does not

notes

- by the end of this module you shall be able to do this

Discussion: around which equilibria may we consider linearizations "good"?

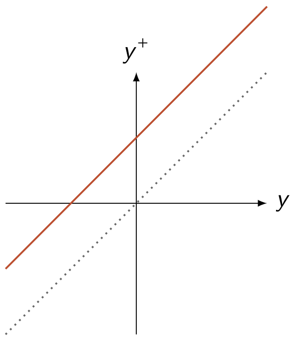


also for the 'unstable' equilibria the approximation may be a good one - depends on the time horizon under consideration and how close y_0 is to the equilibrium

notes

- the fact that A and $\boldsymbol{f}$ are different (even if one is the approximation of the other one) means that, if we think at their physical meaning, starting from the same point the two models give different indications towards where $\boldsymbol{y}$ should go
- this means that the trajectories will be different
- how much different, though? Depends of course on some sort of distance between A and $\boldsymbol{f}$
- for the asymptotically stable equilibrium the approximation will get better and better in time; for the unstable equilibrium worse and worse in time
- recall though that one may consider an arbitrarily small neighborhood of the approximation point. In this way one may think that the linearized version may be an arbitrarily good approximation, if one focuses in a sufficiently small neighborhood
- in this course we will not see how to compute bounds of the error between these two trajectories; you will do it in later on courses

Discussion: is it always meaningful to linearize?

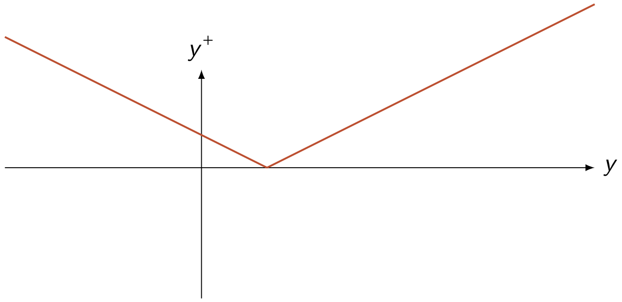


in this case we do not have equilibria

notes

- a simple case that shows that if we do not have equilibria actually we can't linearize

Discussion: and here, can we linearize?

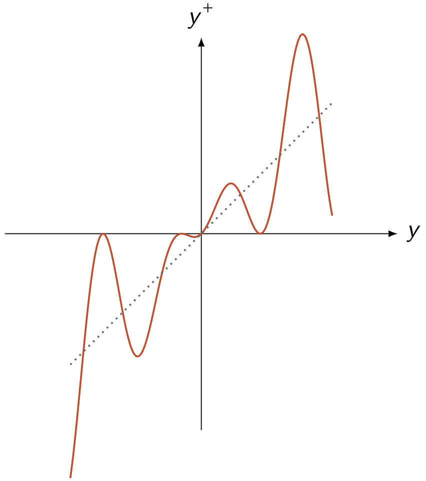


in this case we cannot compute the first derivative

notes

- a simple case that shows that if we do not have continuity for the derivative actually we can't linearize

Discussion: can we trust the stable linearized system for this case?



in this case the basin of attraction is very small

notes

- a simple case that shows that if the basin of attractions are very small, the linearized system may be trusted in a very small region

Summarizing

Assess the validity of the approximation introduced when linearizing a nonlinear RR around an equilibrium point

Evaluate the meaning and applicability of linearization in different contexts, discussing when it provides a reasonable approximation and when it does not

- if we have an asymptotically stable equilibrium, the approximation improves in time
- if we have an unstable equilibrium, the approximation degrades in time
- the closer we start from the equilibrium, the better
- the bigger the curvature of the RR, the more "local" the results will be

notes

- you should now be able to do this, following the pseudo-algorithm in the itemized list

Most important python code for this sub-module

notes

This will do everything for you

https://python-control.readthedocs.io/en/latest/generated/control.linearize.html

though it is dangerous to use tools without knowing how they work

notes

- be always wary of using code without knowing the effects and meaning of the operations they do

Self-assessment material

notes

Question 1

Which of the following statements about linearization around an equilibrium point is correct?

Potential answers:

I: **(wrong)** Linearization provides a good approximation for any nonlinear system at any point.
II: **(wrong)** Linearization is only useful for stable equilibria and does not work for unstable ones.
III: **(correct)** Linearization can be a good approximation near both stable and unstable equilibria, depending on the time horizon and initial conditions.
IV: **(wrong)** Linearization is only valid if the system has no nonlinear terms.
V: **(wrong)** I do not know

Solution 1:

- when is linearizing meaningful 2

Linearization provides a locally valid approximation near an equilibrium point, regardless of stability. However, for unstable equilibria, the validity of the approximation degrades over time as trajectories diverge.

notes

- see the associated solution(s), if compiled with that ones :)

Question 2

Under which condition is linearization not possible?

Potential answers:

I: **(wrong)** If the equilibrium is unstable.
II: **(wrong)** If the equilibrium is stable but far from the origin.
III: **(correct)** If the system's function is not differentiable at the equilibrium point.
IV: **(wrong)** If the system is highly nonlinear.
V: **(wrong)** I do not know

Solution 1:

Linearization relies on computing the first-order Taylor series expansion, which requires differentiability at the equilibrium point. If the function is not differentiable, linearization is not possible.

- when is linearizing meaningful 3

notes

- see the associated solution(s), if compiled with that ones :)

Question 3

How does the curvature of the nonlinear system affect the validity of linearization?

Potential answers:

I: **(wrong)** Curvature does not affect the validity of linearization.
II: **(wrong)** The larger the curvature, the more accurate the linearized model.
III: **(correct)** The larger the curvature, the more local the validity of the linearized model.
IV: **(wrong)** Linearization is only valid when curvature is zero.
V: **(wrong)** I do not know

Solution 1:

Higher curvature means that the nonlinear system deviates more quickly from its linear approximation, making the linearized model valid only in a smaller neighborhood around the equilibrium.

notes

- see the associated solution(s), if compiled with that ones :)

Question 4

What does the size of the basin of attraction tell us about the linearized model?

Potential answers:

I: **(correct)** A small basin of attraction means the linearized model is only valid in a very restricted region.
II: **(wrong)** A small basin of attraction means the system is globally stable.
III: **(wrong)** A large basin of attraction makes linearization unnecessary.
IV: **(wrong)** The basin of attraction does not affect the validity of the linearization.
V: **(wrong)** I do not know

Solution 1:

If the basin of attraction is small, then trajectories quickly leave the region where linearization is valid, limiting its usefulness to a very small neighborhood around the equilibrium.

notes

- see the associated solution(s), if compiled with that ones :)

Question 5

In which of the following cases is linearization not meaningful?

Potential answers:

- I: **(correct)** When the system has no equilibrium points.
- II: **(wrong)** When the equilibrium point is unstable.
- III: **(wrong)** When the system is nonlinear.
- IV: **(wrong)** When the system has high curvature.
- V: **(wrong)** I do not know

Solution 1:

Linearization is performed around an equilibrium point, where the first derivative is evaluated. If no equilibrium exists, linearization is not meaningful.

notes

- see the associated solution(s), if compiled with that ones :)

Recap of sub-module "when is linearizing meaningful"

- be careful when using a linearized system - be always aware of where it comes from

notes

- the most important remarks from this sub-module are these ones