FOUNDATIONS OF SIGNALS AND SYSTEMS
18.3 Homework assignment
Prof. T. Erseghe

Exercises 18.3

Solve the following MatLab problems:

1. Evaluate numerically the Fourier transform of $x(t) = \mathrm{triang}(t)$ and compare it with its analytical expression $X(j\omega) = \mathrm{sinc}^2(\omega/(2\pi))$.

2. The file 'ex18_3_2.mat' contains in vector x pancreatic secretion values taken in the interval $[0, 300]$ min with a sampling spacing of $T = 0.1$ min as well as the impulse response g (on the same time samples) mapping to the plasma concentration $y = x*g$. Plot the signals together with their Fourier transform (absolute values only). Then evaluate the product $X \cdot G$ in the Fourier domain and inverse-transform it by use of the inverse MatLab functions ifftshift and ifft. Compare the result (which is a convolution evaluated in the Fourier domain) with the convolution taken in the time-domain: you should get a perfect correspondence!

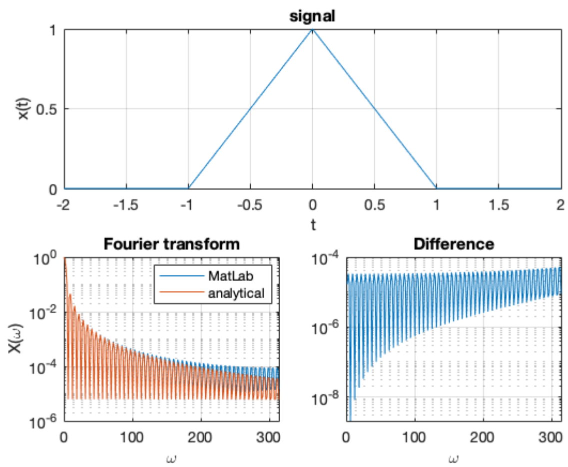
Solutions.

1. The code can mimic that of Exercise 18.2.1, as follows, where we used the time span $[-2, 2]$ for the triangle and a sampling spacing $T = 0.01$. Note that, in this case, we correct the Fourier transform for the fact that the starting sample is not zero. Note also how we show the Fourier transform only in the positive axis where, again, perfect correspondence is available up to $\omega = 100$. In this case we also display the error, showing that it is of the order $2 \cdot 10^{-5}$ along the entire axis, and obviously much more visible where the absolute value of the Fourier transform gets smaller.

```
T = 0.01;
t = -2:T:2;
x = triang(t);
N = length(x);
om = (-round((N-1)/2):round(N/2)-1) *2*pi/(N*T);
X = fftshift(T*fft(x)).*exp(-1i*om*t(1));
Xref = sinc(om/(2*pi)).^2;

figure
subplot(2,1,1)
plot(t,x)
grid
xlabel('t')
ylabel('x(t)')
title('signal')
subplot(2,2,3)
semilogy(om,abs(X),om,abs(Xref))
axis([0 max(om) ylim])
grid
xlabel('\omega')
ylabel('X(\omega)')
legend('MatLab','analytical')
title('Fourier transform')
subplot(2,2,4)
semilogy(om,abs(X-Xref))
axis([0 max(om) ylim])
grid
xlabel('\omega')
title('Difference')

function s = triang(t)
s = (1-abs(t)).*(abs(t)<1);
end
```



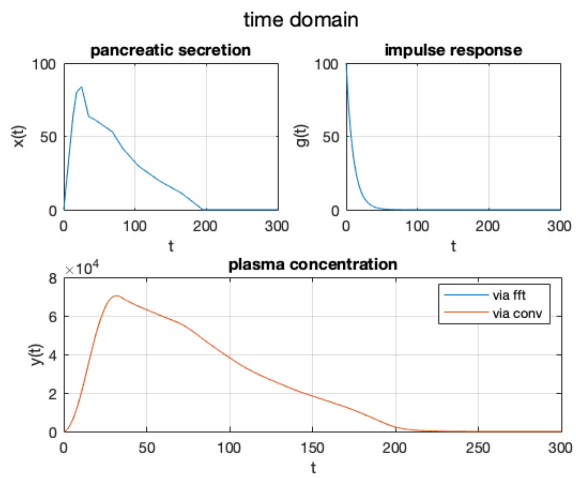
2. In this exercise we first evaluate the Fourier transforms of x and g separately (by using the standard approach), then make a product via the pointwise product operator. The inverse transform is calculated by applying the inverse function ifft and ifftshift in reverse order, to get the correct result. Note the perfect correspondence with the convolution calculated in the time-domain (which is truncated to the same range as x). Incidentally, one could observe that the compact expression "y=T*ifft(fft(x).*fft(g))" holds, where we neglected any use of ω or of the ifftshift operator. This is actually how MatLab calculates convolutions!!!

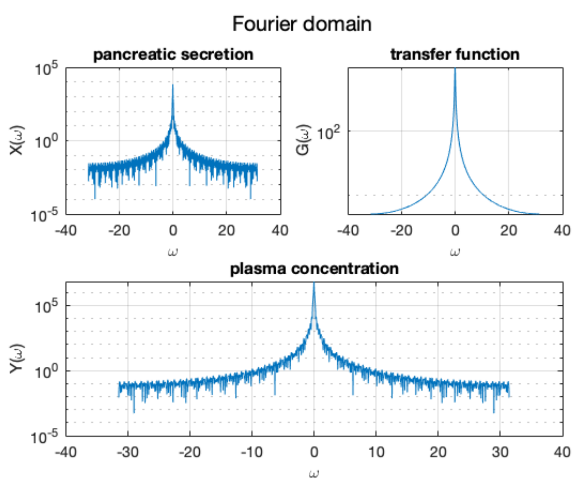
```
load('ex18_3_2.mat') % defines t, x, g, T
N = length(x);
X = fftshift(T*fft(x));
G = fftshift(T*fft(g));
om = (-round((N-1)/2):round(N/2)-1) *2*pi/(N*T);
Y = X.*G;
y = ifft(ifftshift(Y)/T);
y2 = T*conv(x,g);
y2 = y2(1:length(x));

figure(1)
subplot(2,2,1)
plot(t,x)
grid
xlabel('t')
ylabel('x(t)')
title('pancreatic secretion')
subplot(2,2,2)
plot(t,g)
grid
```

```
xlabel('t')
ylabel('g(t)')
title('impulse response')
subplot(2,1,2)
plot(t,y,t,y2)
grid
xlabel('t')
ylabel('y(t)')
legend('via fft','via conv')
title('plasma concentration')
sgtitle('time domain')

figure(2)
subplot(2,2,1)
semilogy(om,abs(X))
grid
xlabel('\omega')
ylabel('X(\omega)')
title('pancreatic secretion')
subplot(2,2,2)
semilogy(om,abs(G))
grid
xlabel('\omega')
ylabel('G(\omega)')
title('transfer function')
subplot(2,1,2)
semilogy(om,abs(Y))
grid
xlabel('\omega')
ylabel('Y(\omega)')
title('plasma concentration')
sgtitle('Fourier domain')
```


time domain
pancreatic secretion
impulse response
x(t)
g(t)
t
plasma concentration
y(t)
×10^4
via fft
via conv


Fourier domain
pancreatic secretion
transfer function
X(ω)
G(ω)
ω
plasma concentration
Y(ω)