Le30

Tuesday, 3 June 2025 14:50

ES1 $x(n) = y(n-2) + y(n-1) - 6\,y(n)$

$x(n) = A$

$y(-1) = K_1$, $y(-2) = K_2$

① $h(n) = ?$ BIBO STABILE?

② $y(n) = ?$ $n \geq 0$

①

$$H(z) = \frac{b(z)}{a(z)} = \frac{1}{z^{-2} + z^{-1} - 6\,\cancel{z^0}} = \frac{1}{(z^{-1}-2)(z^{-1}+3)}$$

$$z^{-1}_{1,2} = \frac{-1 \pm \sqrt{1+24}}{2} = \begin{cases} 2 \\ -3 \end{cases}$$

NON SONO I POLI

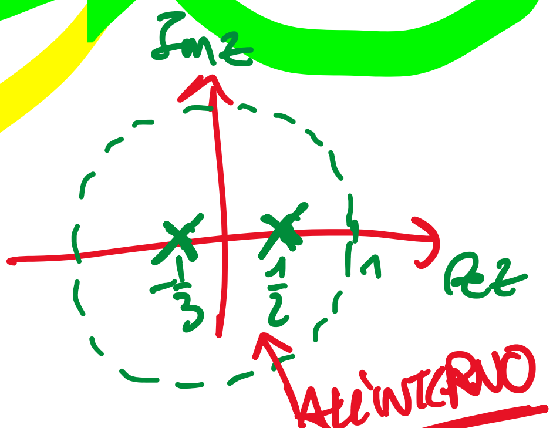
$$H(z) = \frac{1}{(z^{-1}-2)(z^{-1}+3)} = \frac{R_0}{z^{-1}-2} + \frac{R_1}{z^{-1}+3}$$

$$P_{1,2} = \begin{cases} 1/2 \\ -1/3 \end{cases}$$

$$R_0 = \frac{1}{z^{-1}+3}\Big|_{z^{-1}=2} = \frac{1}{5}$$

$$R_1 = \frac{1}{z^{-1}-2}\Big|_{z^{-1}=-3} = -\frac{1}{5}$$

$$\frac{1}{z^{-1} - P_0^{-1}} \xrightarrow{\mathcal{Z}^{-1}} -P_0^{n+1}\,1_0(n)$$



ALL'INTERNO

BIBO STABILE $|P_{1,2}| < 1$

$$h(n) = \frac{1}{5} \cdot -\left(\frac{1}{2}\right)^{n+1} 1_0(n) - \frac{1}{5} \cdot -\left(-\frac{1}{3}\right)^{n+1} 1_0(n)$$

$$= -\frac{1}{5}\left(\frac{1}{2}\right)^{n+1} 1_0(n) + \frac{1}{5} \cdot \left(-\frac{1}{3}\right)^{n+1} 1_0(n)$$

②

$$x(n) = y(n-2) + y(n-1) - 6\,y(n)$$

$$\downarrow \mathcal{Z} \qquad \downarrow \mathcal{Z} \qquad \downarrow \mathcal{Z} \qquad \downarrow \mathcal{Z}$$

$$X(z) = z^{-2}Y(z) + z^{-1}Y(z) - 6\,Y(z)$$

$$+ z^{-1}\cancel{y(-1)}K_1 \quad + \cancel{y(-1)}K_1$$

$$+ \cancel{y(-2)}K_2$$

$$Y(z)\left(\cancel{z^{-2} + z^{-1} - 6}\right) = \frac{X(z) - \left(z^{-1}K_1 + K_1 + K_2\right)}{z^{-2}+z^{-1}-6}$$

$$Y(z) = \underbrace{\frac{1}{(z^{-1}-2)(z^{-1}+3)}\,X(z)}_{Y_f(z)} - \underbrace{\frac{z^{-1}K_1 + K_1 + K_2}{(z^{-1}-2)(z^{-1}+3)}}_{Y_e(z)}$$

$x(n) = A$

$$x_+(n) = A \cdot 1_0(n) \longrightarrow X(z) = \frac{-A}{z^{-1}-1}$$

$$+\cancel{P_0}^{n+1} 1_0(n) \longrightarrow \frac{-1}{z^{-1} - \cancel{P_0}1} \qquad P_0 = 1$$

$$Y_f(z) = \frac{-A}{(z^{-1}-2)(z^{-1}+3)(z^{-1}-1)} = \frac{R_0}{z^{-1}-2} + \frac{R_1}{z^{-1}+3} + \frac{R_2}{z^{-1}-1}$$

$$R_0 = \frac{-A}{(z^{-1}+3)(z^{-1}-1)}\Big|_{z^{-1}=2} = -\frac{A}{5}$$

$$R_1 = \frac{-A}{(z^{-1}-2)(z^{-1}-1)}\Big|_{z^{-1}=-3} = -\frac{A}{20}$$

$$R_2 = \frac{-A}{(z^{-1}-2)(z^{-1}+3)}\Big|_{z^{-1}=1} = \frac{A}{4}$$

$$y_f(n) = +\frac{A}{5} \cdot +\left(\frac{1}{2}\right)^{n+1} 1_0(n) + \frac{A}{20} \cdot +\left(-\frac{1}{3}\right)^{n+1} 1_0(n)$$

$$+ \frac{A}{4} \cdot -\cancel{(1)^{n+1}}\, 1_0(n)$$

ES2 $h(n) = (1+2n)(-1)^n 1_0(n) + \frac{1}{2}\left(-\frac{1}{2}\right)^n 1_0(n)$

$$x(n) = \frac{1}{3}\left(-\frac{1}{3}\right)^n 1_0(n) = -\left(-\frac{1}{3}\right)^{n+1} 1_0(n) \xrightarrow{\mathcal{Z}} X(z) = \frac{1}{z^{-1}+3}$$

COND. INIZIALI NULLE

1) BIBO STABILE **NO**

2) EQ. DIFFERENZE → MI SERVE $H(z)$

3) EV LIBERA + RISP. FORZATA

$y_e(n) = 0$

$$(n+1)\,P_0^{n+2}\,1_0(n) \longrightarrow \frac{1}{(z^{-1} - P_0^{-1})^2}$$

$$-P_0^{n+1}\,1_0(n) \longrightarrow \frac{1}{z^{-1} - P_0^{-1}}$$

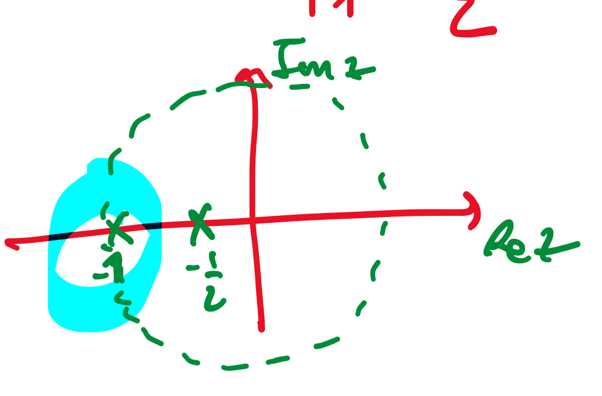
$$h(n) = \underbrace{(1+2n)}_{2(n+1)-1} (-1)^n 1_0(n) - \left(-\frac{1}{2}\right)^{n+1} 1_0(n)$$

$$= 2(n+1)(-1)^{n+2} 1_0(n) + (-1)^{n+1} 1_0(n) - \left(-\frac{1}{2}\right)^{n+1} 1_0(n)$$

$$H(z) = \frac{2}{(z^{-1}+1)^2} - \frac{1}{z^{-1}+1} + \frac{1}{z^{-1}+2}$$

$$= \frac{2(z^{-1}+2) - (z^{-1}+1)\cancel{(z^{-1}+2)} + \cancel{(z^{-1}+1)^2}}{(z^{-1}+1)^2(z^{-1}+2)}$$

$$H(z) = \frac{z^{-1}+3}{(z^{-1}+1)^2(z^{-1}+2)} \longrightarrow \text{POLI} \quad P_0 = -1, \quad P_1 = -\frac{1}{2}$$



$$(z^{-2} + 2z^{-1} + 1)(z^{-1}+2) =$$

$$z^{-3} \; \overset{4}{+\cancel{2}} z^{-2} \; \cancel{+z^{-1}}$$

$$\cancel{+2z^{-2}} \; \underset{5}{+\cancel{4}} z^{-1} \; +2$$

$$x(n-1) + 3x(n) = y(n-3) + 4\,y(n-2) + 5\,y(n-1) + 2\,y(n)$$

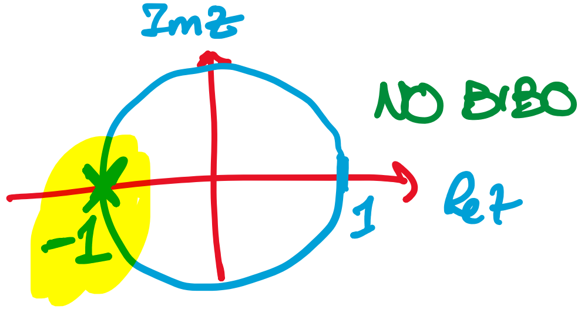
$$Y_f(z) = H(z)\,X(z) = \frac{\cancel{z^{-1}+3}}{(z^{-1}+1)^2(z^{-1}+2)} \cdot \frac{1}{\cancel{z^{-1}+3}}$$

$$= \frac{R_0}{(z^{-1}+1)^2} + \frac{R_1}{(z^{-1}+1)} + \frac{R_2}{z^{-1}+2}$$

$$R_0 = \frac{1}{z^{-1}+2}\Big|_{z^{-1}=-1} = 1$$

$$R_1 = \frac{\partial}{\partial x}\left(\frac{1}{x+2}\right)\Big|_{x=-1} = \frac{-1}{(x+2)^2}\Big|_{x=-1} = -1$$

NOTA $H(z) = \frac{1}{z^{-1}+1}$ $\quad h(n) = (-1)^n 1_0(n)$



NO BIBO STABILE

TROVARE $x(n)$ LIMITATO CHE DÀ $y_f(n)$ NON LIMITATA

$$X(z) = \frac{1}{z^{-1} - P_0^{-1}}$$

$$Y_f(z) = H(z)X(z) = \frac{1}{(z^{-1}+1)(z^{-1}-P_0^{-1})} = \frac{R_0}{z^{-1}+1} + \frac{R_1}{z^{-1}-P_0^{-1}}$$

$$y_f(n) = \alpha\,(-1)^n 1_0(n) + B\,(P_0)^n 1_0(n)$$

$|P_0| \leq 1$

CASO $P_0 \neq -1$

caso $P_0 = -1$

$$Y_f(z) = \frac{1}{(z^{-1}+1)^2} \xrightarrow{\mathcal{Z}^{-1}} (n+1)(-1)^{n+2} 1_0(n)$$

NON LIMITATA