Le29

Tuesday, 3 June 2025 10:29

ES1

$$Y(s)=\underbrace{\frac{1/m}{s^2+K/m}\cdot X(s)}_{Y_f(s)}+\underbrace{\frac{v_0+sy_0}{s^2+K/m}}_{Y_\ell(s)\ \text{EVOL. LIBERA}}$$

$$x(t)=F_0\cos(\omega_0 t)\,1(t)$$

$$y_f(t)=?$$

$$\boxed{\begin{aligned}\cos(\omega_0 t)\,1(t)&\xrightarrow{\mathcal{L}}\frac{s}{s^2+\omega_0^2}\\ \sin(\omega_0 t)\,1(t)&\xrightarrow{\mathcal{L}}\frac{\omega_0}{s^2+\omega_0^2}\end{aligned}}$$

$$X(s)=\frac{F_0 s}{s^2+\omega_0^2}$$

$$Y_\ell(s)=\frac{v_0}{\sqrt{K/m}}\cdot\frac{\sqrt{K/m}}{s^2+K/m}+y_0\frac{s}{s^2+K/m}$$

$$y_\ell(t)=\sqrt{\frac{m}{K}}\,v_0\sin\left(\sqrt{\frac{K}{m}}\,t\right)1(t)+y_0\cos\left(\sqrt{\frac{K}{m}}\,t\right)1(t)$$

$$Y_f(s)=\frac{F/m\cdot s}{(s^2+K/m)(s^2+\omega_0^2)}$$

$$\omega_0^2\neq\frac{K}{m}$$

POLI $\pm j\omega_0$ $\pm j\sqrt{\frac{K}{m}}$

$$Z(s^2)=\frac{1}{(s^2+K/m)(s^2+\omega_0^2)}=\frac{R_0}{s^2+\omega_0^2}+\frac{R_1}{s^2+K/m}$$

$$R_0=Z(s^2)(s^2+\omega_0^2)\Big|_{s^2=-\omega_0^2}=\frac{1}{s^2+\frac{K}{m}}\Big|_{s^2=-\omega_0^2}$$

$$=\frac{1}{\frac{K}{m}-\omega_0^2}$$

$$R_1=Z(s^2)(s^2+K/m)\Big|_{s^2=-\frac{K}{m}}=\frac{1}{s^2+\omega_0^2}\Big|_{s^2=-K/m}$$

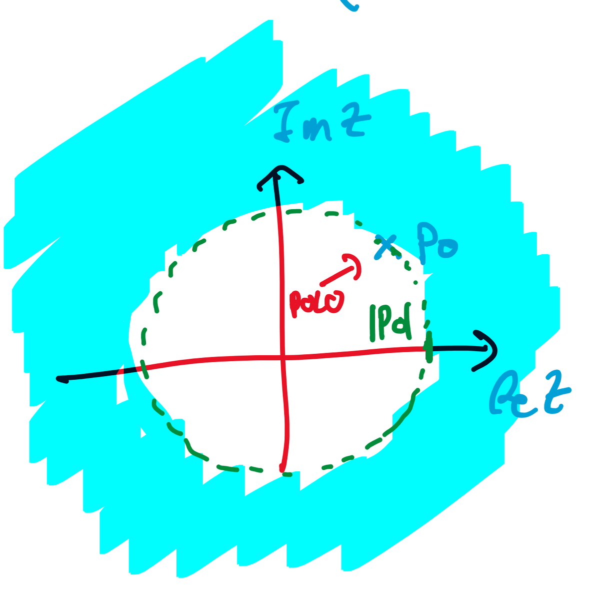
$$=\frac{1}{-\frac{K}{m}+\omega_0^2}=-R_0$$

$$Y_f(s)=\frac{F_0/\cancel{m}\,\cancel{s}\cdot\cancel{R_0}}{\frac{K}{\cancel{m}}-\omega_0^2 m}\left(\frac{s}{s^2+\omega_0^2}-\frac{s}{s^2+K/m}\right)$$

$$y_f(t)=\frac{F_0}{K-\omega_0^2 m}\left(\cos(\omega_0 t)-\cos\left(\sqrt{\frac{K}{m}}\,t\right)\right)1(t)$$

ES2 TROVARE $X(z)$ PER $x(n)=p_0^n 1_0(n)$ $p_0\in\mathbb{C}$

$$X_+(z)=\sum_{n=0}^{+\infty}\underbrace{p_0^n\,\cancel{1_0(n)}\cdot z^{-n}}_{(p_0z^{-1})^n}=\frac{1}{1-p_0z^{-1}}=\frac{z}{z-p_0}$$



$$|p_0z^{-1}|<1$$

$$\frac{|p_0|}{|z|}<1$$

$$\boxed{|p_0|<|z|}$$

ES3 TROVARE $X(z)$ PER $x(n)=\cos(\theta_0 n)\,1_0(n)$

$$X_+(z)=\sum_{n=0}^{\infty}\underbrace{\cos(\theta_0 n)}_{\frac{1}{2}e^{j\theta_0 n}+\frac{1}{2}e^{-j\theta_0 n}=\frac{1}{2}(e^{j\theta_0})^n+\frac{1}{2}(e^{-j\theta_0})^n}\cancel{1_0(n)}\,z^{-n}$$

$$x(n)=\frac{1}{2}(e^{j\theta_0})^n 1_0(n)+\frac{1}{2}(e^{-j\theta_0})^n 1_0(n)$$

$$X_+(z)=\frac{1}{2}\,\frac{1}{1-e^{j\theta_0}z^{-1}}+\frac{1}{2}\,\frac{1}{1-e^{-j\theta_0}z^{-1}}$$

$$|z|>|e^{\pm j\theta_0}|=1$$

$$=\frac{1-e^{-j\theta_0}z^{-1}+1-e^{j\theta_0}z^{-1}}{2(1-e^{j\theta_0}z^{-1})(1-e^{-j\theta_0}z^{-1})}$$

$$=\frac{1-z^{-1}\overbrace{\frac{e^{j\theta_0}+e^{-j\theta_0}}{2}}^{\cos\theta_0}}{1-z^{-1}\underbrace{(e^{j\theta_0}+e^{-j\theta_0})}_{2\cos\theta_0}+\cancel{e^{j\theta_0}}\cancel{e^{-j\theta_0}}z^{-2}}$$

$$=\frac{1-\cos\theta_0\,z^{-1}}{1-2\cos\theta_0\,z^{-1}+z^{-2}}$$

ES3 TROVARE $X(z)$ PER $x(n)=-p_0^{n+1}1_0(n)$

$$x(n)=-p_0\cdot p_0^n 1_0(n)$$

$$X_+(z)=-p_0\cdot\frac{1}{1-p_0z^{-1}}$$

$$|z|>|p_0|$$

$$=\frac{1}{z^{-1}-\frac{1}{p_0}}$$

NOTA $\frac{1}{z^{-1}-3}$ ← POLO $p_0=\frac{1}{3}$

$$\downarrow Z^{-1}$$

$$-\left(\frac{1}{3}\right)^{n+1}1_0(n)$$

ES4 TROVARE $X(z)$ PER $x(n)=(n+1)\,p_0^{n+2}1_0(n)$

$$x(n)=p_0\left(n\cdot p_0^{n+1}1_0(n)+p_0^{n+1}1_0(n)\right)$$

$$X_+(z)=p_0\left(+\cancel{z}\,\underbrace{Y'(z)}_{\frac{+1\cdot+1\cdot+z^{-1}}{(z^{-1}-p_0^{-1})^2}}\ \underbrace{-\frac{1}{z^{-1}-p_0^{-1}}}_{Y(z)}\right)$$

$$=p_0\left(\frac{\cancel{z^{-1}}}{(z^{-1}-p_0^{-1})^2}+\frac{\cancel{-z^{-1}}+p_0^{-1}}{(z^{-1}-p_0^{-1})^2}\right)$$

$$=\frac{1}{(z^{-1}-p_0^{-1})^2}$$

$$\boxed{\begin{aligned}(n+1)\,p_0^{n+2}1_0(n)&\xrightarrow{Z}\frac{1}{(z^{-1}-p_0^{-1})^2}\\ -p_0^{n+1}1_0(n)&\xrightarrow{Z}\frac{1}{(z^{-1}-p_0^{-1})}\end{aligned}}$$

$$-\frac{(n+1)(n+2)}{2}\,p_0^{n+3}1_0(n)\xrightarrow{Z}\frac{1}{(z^{-1}-p_0^{-1})^3}$$