Le28

Friday, 30 May 2025 10:27

ES1 FILTRO RC

$y(t) + RC\,y'(t) = x(t)$

$x(t) = A$

$y(0^-) = V_0$

$y(t) = ?\ \ t > 0$

$y_+(t) + RC\,y_+'(t) = x_+(t)$

$\downarrow \mathcal{L} \qquad \downarrow \mathcal{L} \qquad \downarrow \mathcal{L}$

$Y(s) + RC\left(sY(s) - y(0^-)\right) = X(s)$

$Y(s)\left(1 + RCs\right) - RC\,V_0 = X(s)$

$Y(s) = \dfrac{X(s)}{1+RCs} + \dfrac{RC\,V_0}{1+RCs}$

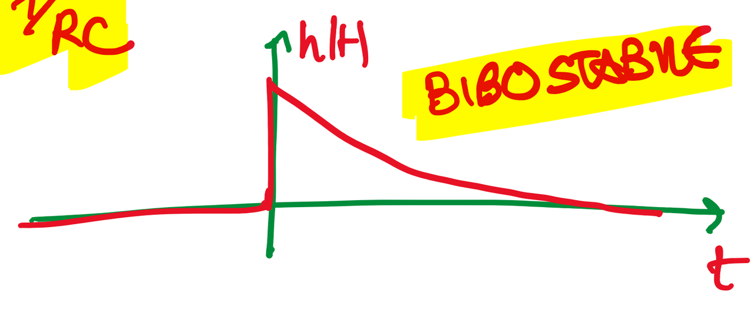
RISP. FORZATA — EVOLUZIONE LIBERA

$Y(s) = H(s)X(s) + \dfrac{V_0}{s + 1/RC} \xrightarrow{\mathcal{L}^{-1}} y_n(t) = V_0 e^{-t/RC}\,1(t)$

$H(s) = \dfrac{1}{1+RCs} = \dfrac{1/RC}{s + 1/RC} \xrightarrow{\mathcal{L}^{-1}} h(t) = \dfrac{1}{RC} e^{-\frac{1}{RC} t}\,1(t)$

$P_1 = -\dfrac{1}{RC}$

$Re[P_1] < 0$



BIBO STABILE

$Y_f(s) = H(s)X(s) = \dfrac{1/RC}{s + 1/RC} \cdot \dfrac{A}{s} = \dfrac{A/RC}{s(s + 1/RC)}$

$\uparrow \mathcal{L}$

$x_f(t) = A \cdot 1(t)$

$Y_f(s) = \dfrac{A/RC}{s(s + 1/RC)} = \dfrac{R_0}{s} + \dfrac{R_1}{s + 1/RC}$

$R_0 = Y_f(s) \cdot s \Big|_{s=0} = \dfrac{A/RC}{s + 1/RC}\Big|_{s=0} = A$

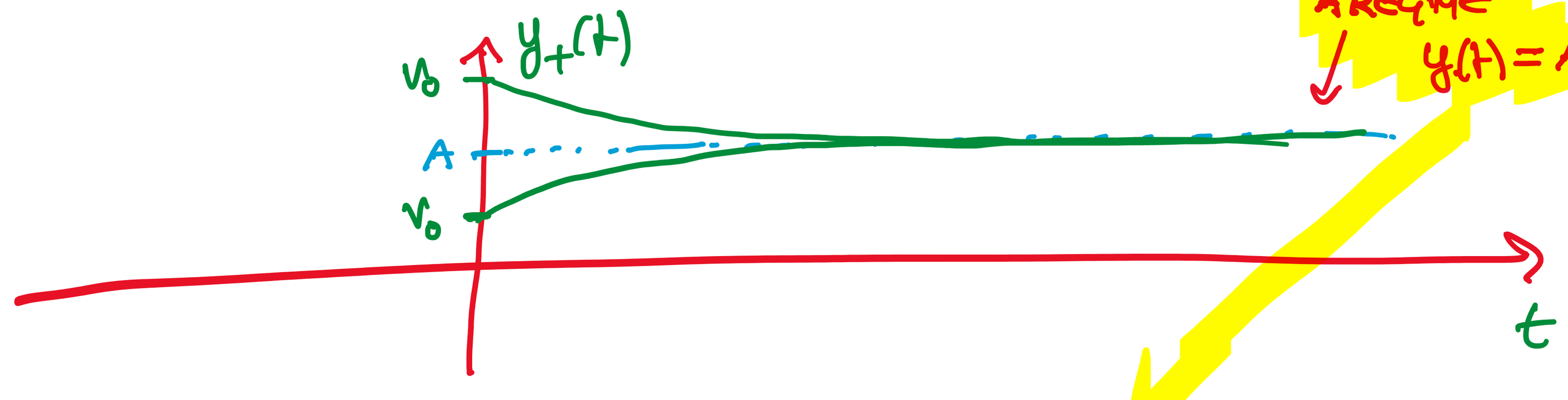
$R_1 = Y_f(s)\,(s + 1/RC)\Big|_{s=-\frac{1}{RC}} = \dfrac{A/RC}{s}\Big|_{s=-\frac{1}{RC}} = -A$

$Y_f(s) = \dfrac{A}{s} - \dfrac{A}{s + \frac{1}{RC}} \xrightarrow{\mathcal{L}^{-1}} A \cdot 1(t) - A e^{-t/RC}\,1(t) = y_f(t)$

$y_+(t) = y_f(t) + y_n(t)$

$= A \cdot 1(t) - A e^{-t/RC}\,1(t) + V_0 e^{-t/RC}\,1(t)$

$= A \cdot 1(t) + (V_0 - A)\, e^{-t/RC}\,1(t)$



A REGIME $y(t) = A$

$x(t) = A \rightarrow \boxed{h(\cdot)} \rightarrow y(t) = H(j0) \cdot A = A \qquad t \gg 0$

$H(s) = \dfrac{1}{1 + sRC}\Big|_{s=0} = 1$

ES2

$y''(t) - y'(t) - 6y(t) = x'(t) - 3x(t)$

1) $H(s)$
2) BIBO STABILE
3) RISPOSTA FORZATA CON $x(t) = 1(t)$
4) SIA $x(t) = A\cos(\omega_0 t + \theta_0)$ E CONDIZIONI INIZIALI NULLE TROVARE ω_0 CHE GARANTISCA UN COMPORTAMENTO A REGIME $y(t) = \frac{1}{5}x(t - t_0)$ PER $t \gg 0$

$H(s) = \dfrac{b(s)}{a(s)} = \dfrac{s-3}{s^2 - s - 6} = \dfrac{s-3}{(s-3)(s+2)} = \dfrac{1}{s+2}$ BIBO STABILE

$P_{1,2} = \dfrac{1 \pm \sqrt{1+24}}{2} = 3,\ -2$

$x(t) = y'(t) + 2y(t)$

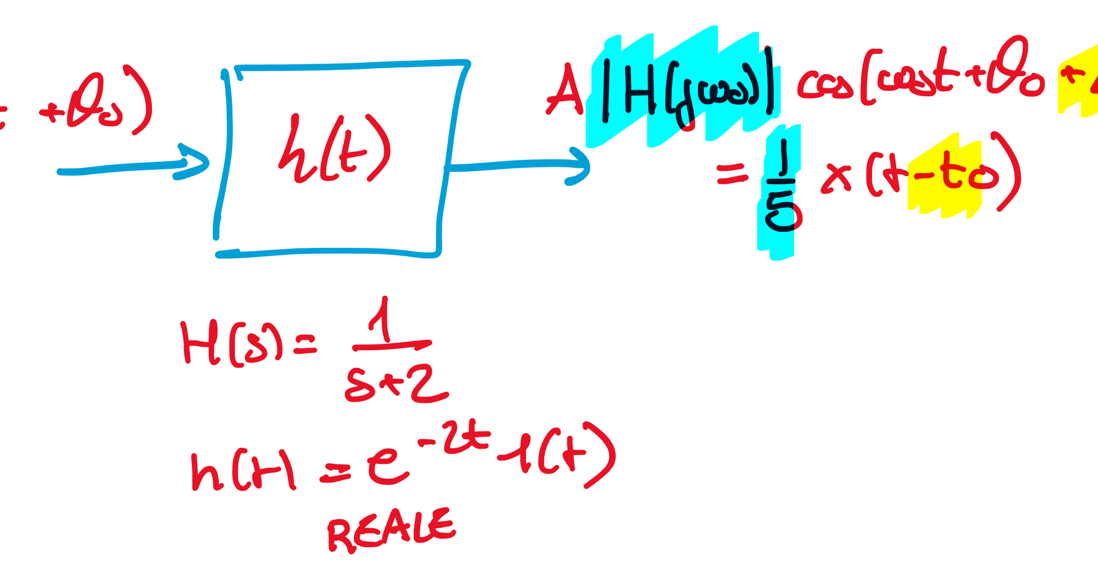
EQ. DIFFERENZIALE EQUIVALENTE

$Y_f(s) = H(s)X(s) = \dfrac{1}{s+2} \cdot \dfrac{1}{s} = \dfrac{R_0}{s} + \dfrac{R_1}{s+2}$

$R_0 = Y_f(s)\,s\big|_{s=0} = \dfrac{1}{s+2}\Big|_{s=0} = 1/2$

$R_1 = Y_f(s)\,(s+2)\big|_{s=-2} = \dfrac{1}{s}\Big|_{s=-2} = -1/2$

$y_f(t) = \frac{1}{2}\,1(t) - \frac{1}{2} e^{-2t}\,1(t)$



A REGIME

$x(t) = A\cos(\omega_0 t + \theta_0) \rightarrow \boxed{h(t)} \rightarrow A\,|H(j\omega_0)| \cos(\omega_0 t + \theta_0 + \angle H(j\omega_0)) = \frac{1}{5}\,x(t - t_0)$

$H(s) = \dfrac{1}{s+2}$

$h(t) = e^{-2t}\,1(t)$ REALE

$|H(j\omega_0)| = \dfrac{1}{5} = \left|\dfrac{1}{2 + j\omega_0}\right|$

$|2 + j\omega_0|^2 = 5^2 \rightarrow 4 + \omega_0^2 = 25 \quad (21)$

$\omega_0 = \pm\sqrt{21}$

ES3

$x(t) = K y(t) + m y''(t)$

$x(t) = F_0 \cos(\omega_0 t)$

$y(0_-) = y_0$

$y'(0_-) = v_0$

$y(t) = ?\ \ t \geq 0$

$x_+(t) = K\,y_+(t) + m\,y_+''(t)$

$\downarrow \mathcal{L} \qquad \downarrow \mathcal{L} \qquad \downarrow \mathcal{L}$

$X(s) = K Y(s) + m\left(s^2 Y(s) - s\,y(0^-) - y'(0^-)\right)$ (con $y(0^-) = y_0$, $y'(0^-) = v_0$)

$\downarrow$

$Y(s)\left(K + ms^2\right) - m\left(s y_0 + v_0\right) = X(s)$

$\downarrow$

$Y(s) = \dfrac{X(s)}{K + ms^2} + \dfrac{m(s y_0 + v_0)}{K + ms^2}$

$Y(s) = \underbrace{\dfrac{1/m}{s^2 + K/m}}_{H(s)} \cdot X(s) + \dfrac{v_0 + s y_0}{s^2 + K/m}$

POLI $\pm j\sqrt{K/m}$ NON BIBO STABILE!

$h(t) = \dfrac{1/m}{\sqrt{K/m}} \sin\left(\sqrt{\dfrac{K}{m}}\,t\right) 1(t) \qquad \dfrac{1/m}{\sqrt{K/m}} = \dfrac{1}{\sqrt{Km}}$

$\cos(\omega_0 t)\,1(t) \rightarrow \dfrac{s}{s^2 + \omega_0^2}$

$\sin(\omega_0 t)\,1(t) \rightarrow \dfrac{\omega_0}{s^2 + \omega_0^2}$