Le27

Friday, 23 May 2025 08:49

NOTA

$$H(s) = \frac{R_1}{(s-p_1)} + \sum_k \frac{R_k}{(s-p_k)} \qquad p_1 \neq p_k$$

$$H(s)(s-p_1)\Big|_{s=p_1} = \frac{R_1}{\cancel{(s-p_1)}}\cancel{(s-p_1)} + \underbrace{\sum_k \frac{R_k}{\underset{\neq 0}{(s-p_k)}} \overset{=0}{(s-p_1)}}_{0}\Big|_{s=p_1} = R_1$$

ES1 TROVARE $x(t)$ CAUSALE PER $X(s) = \frac{s-3}{s+2}$ FRAZIONE IMPROPRIA

$$\underset{b(s)}{(s-3)} : \underset{a(s)}{(s+2)} = \underset{q(s)}{1}$$

$$\begin{array}{r} s-3 \\ -s-2 \\ \hline 0\ \ -5 \ \ r(s) \end{array}$$

$$X(s) = \frac{b(s)}{a(s)} = \frac{a(s)q(s)+r(s)}{a(s)} = q(s) + \frac{r(s)}{a(s)}$$

$$= 1 - \frac{5}{s+2} = \frac{s+2-5}{s+2} = \frac{s-3}{s+2} \checkmark$$

$$\delta(t) \xrightarrow{\mathcal{L}} 1$$

$$e^{p_0 t}1(t) \xrightarrow{\mathcal{L}} \frac{1}{s-p_0} \qquad p_0 = -2$$

$$x(t) = \delta(t) - 5e^{-2t}1(t)$$

ES2 TROVARE L'ANTITRASFORMATA CAUSALE DI $X(s) = \frac{1}{s^3+s^2-6s}$ FRAZIONE PROPRIA

$$X(s) = \frac{1}{s(s^2+s-6)} = \frac{1}{s(s-2)(s+3)}$$

$$p_{1,2} = \frac{-1 \pm \sqrt{1+24}}{2} = \frac{-1\pm 5}{2} = \begin{cases} 2 \\ -3 \end{cases}$$

$$X(s) = \frac{1}{s(s-2)(s+3)} = \frac{R_0}{s} + \frac{R_1}{s-2} + \frac{R_2}{s+3}$$

$p_0=0 \quad p_1=2 \quad p_2=-3$

$$R_0 = X(s)\cdot s\Big|_{s=0} = \frac{1}{\cancel{s}(s-2)(s+3)}\cancel{s}\Big|_{s=0} = -\frac{1}{6}$$

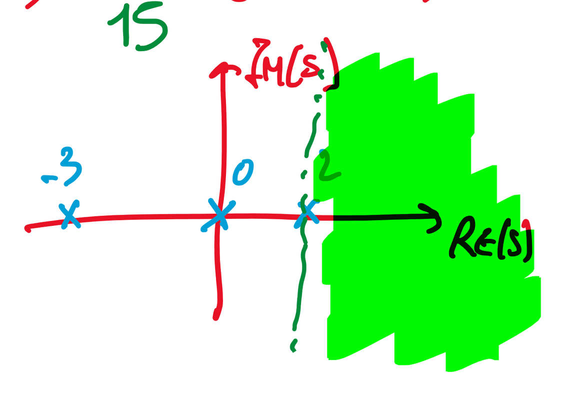
$$R_1 = X(s)(s-2)\Big|_{s=2} = \frac{1}{s\cancel{(s-2)}(s+3)}\cancel{(s-2)}\Big|_{s=2} = \frac{1}{10}$$

$$R_2 = X(s)(s+3)\Big|_{s=-3} = \frac{1}{s(s-2)\cancel{(s+3)}}\cancel{(s+3)}\Big|_{s=-3} = \frac{1}{15}$$

$$X(s) = -\frac{1}{6}\cdot\frac{1}{s} + \frac{1}{10}\cdot\frac{1}{s-2} + \frac{1}{15}\cdot\frac{1}{s+3}$$

$\mathcal{L}^{-1}\downarrow$

$$x(t) = -\frac{1}{6}1(t) + \frac{1}{10}e^{2t}1(t) + \frac{1}{15}e^{-3t}1(t)$$



NOTA

$$X(s) = \frac{R_0}{s-p_0} + \frac{R_1}{(s-p_0)^2} + \frac{R_2}{(s-p_0)^3} + \sum_k \frac{R_k}{(s-p_k)^{m_k}} \qquad p_0 \neq p_k$$

MOLTEPLICITÀ 3

$$Y(s) = X(s)(s-p_0)^3 = R_0(s-p_0)^2 + R_1(s-p_0) + R_2 + \sum_k R_k\frac{(s-p_0)^3}{(s-p_k)^{m_k}}$$

$$Y(s)\Big|_{s=p_0} = 0 + 0 + R_2 + 0 = R_2$$

$$Y'(s) = 2R_0(s-p_0) + R_1 + 0 + \sum_k R_k\left[\frac{3(s-p_0)^2}{(s-p_k)^{m_k}} - m_k\frac{(s-p_0)^3}{(s-p_k)^{m_k+1}}\right]$$

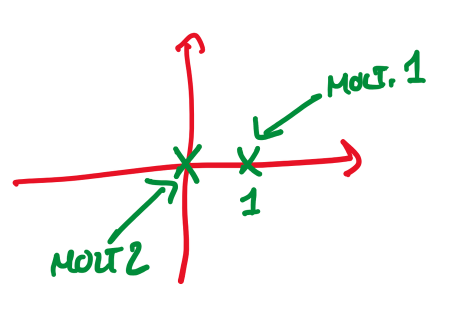
$$Y'(s)\Big|_{s=p_0} = 0 + R_1 + 0 + 0 = R_1$$

$$\frac{1}{2}Y''(s) = R_0 + 0 + 0 + \sum_k \frac{R_k}{2}\left[\frac{6(s-p_0)}{(s-p_k)^{m_k}} - \frac{3m_k(s-p_0)^2}{(s-p_k)^{m_k+1}} - \frac{3m_k(s-p_0)^2}{(s-p_k)^{m_k+1}} + \frac{m_k(m_k+1)(s-p_0)^3}{(s-p_k)^{m_k+2}}\right]$$

$$\frac{1}{2}Y''(s)\Big|_{s=p_0} = R_0 + 0 + 0 + 0 = R_0$$

ES3 TROVARE L'ANTITRASFORMATA CAUSALE DI $X(s) = \frac{4s-1}{2s^2(s-1)}$

$$X(s) = \frac{4s-1}{2s^2(s-1)} = \frac{2s-\frac{1}{2}}{s^2(s-1)} = \frac{R_0}{s} + \frac{R_1}{s^2} + \frac{R_2}{s-1}$$



$$R_2 = X(s)(s-1)\Big|_{s=1} = \frac{2s-\frac{1}{2}}{s^2}\Big|_{s=1} = \frac{3}{2}$$

$$R_1 = \underbrace{X(s)s^2}_{Y(s)}\Big|_{s=0} = \underbrace{\frac{2s-\frac{1}{2}}{s-1}}_{Y(s)}\Big|_{s=0} = \frac{1}{2}$$

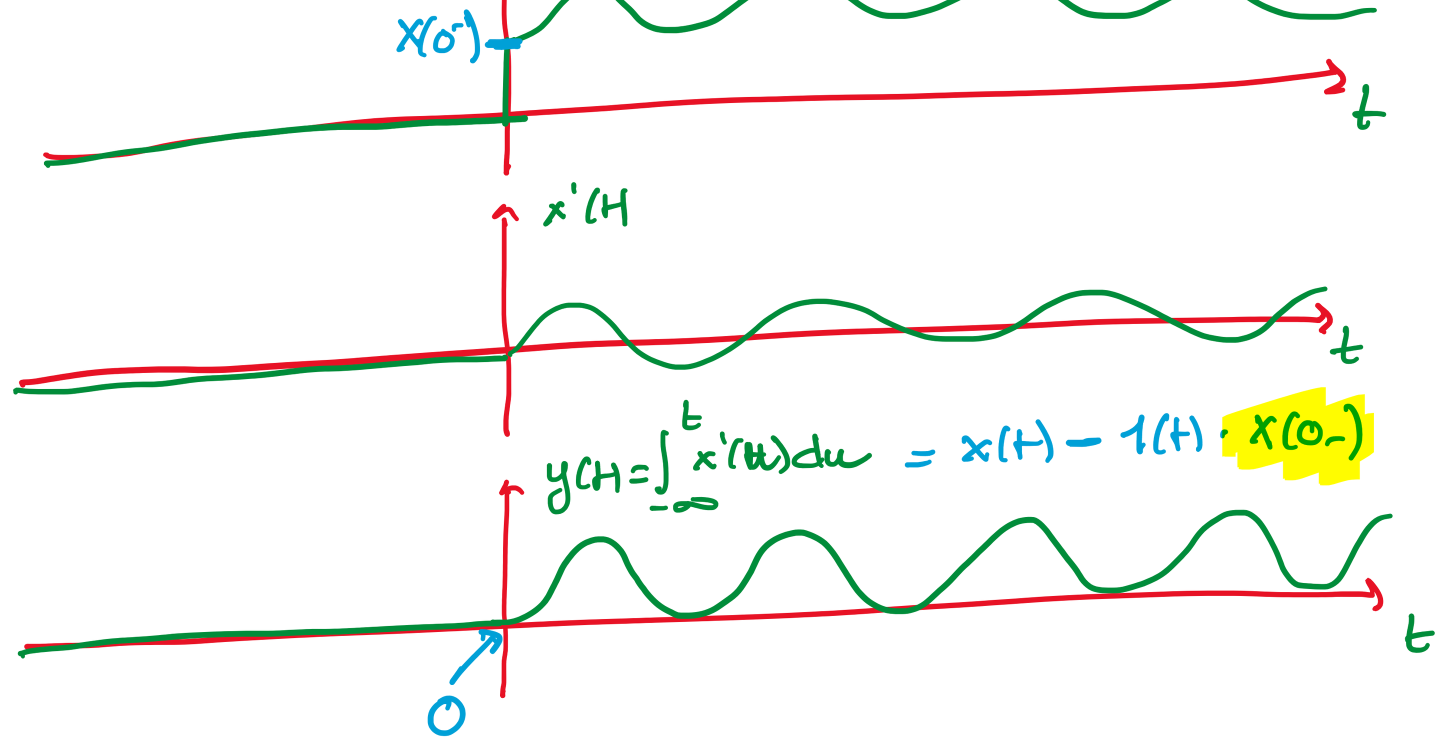
$$R_0 = Y'(s)\Big|_{s=0} = \frac{2}{s-1} - \frac{2s-\frac{1}{2}}{(s-1)^2}\Big|_{s=0} = -2 + \frac{1}{2} = -\frac{3}{2}$$

$$X(s) = -\frac{3}{2}\cdot\frac{1}{s} + \frac{1}{2}\cdot\frac{1}{s^2} + \frac{3}{2}\cdot\frac{1}{s-1}$$

$\mathcal{L}^{-1}\downarrow$

$$x(t) = -\frac{3}{2}1(t) + \frac{1}{2}t\cdot 1(t) + \frac{3}{2}e^{t}\cdot 1(t)$$

NOTA



$$y(t) = \int_{-\infty}^{t} x'(u)\,du = x(t) - 1(t)\cdot x(0_-)$$