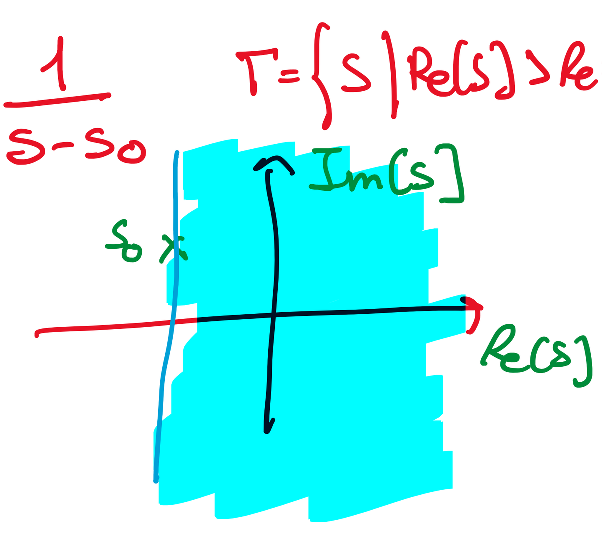
Le25

Thursday, 22 May 2025 08:45

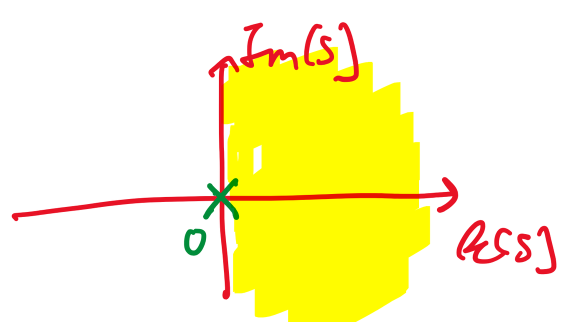
NOTA $e^{s_0 t} 1(t) \xrightarrow{\mathcal{L}} \dfrac{1}{s-s_0}$ $\Gamma = \{ s \mid Re[s] > Re[s_0] \}$



ES1 TROVARE $X(s)$ PER $x(t) = 1(t)$

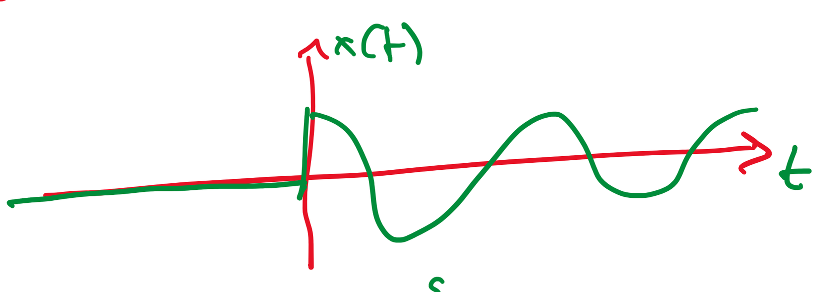
PER $s_0 = 0$

$e^{0 \cdot t} 1(t) = 1(t) \xrightarrow{\mathcal{L}} \dfrac{1}{s-0} = \dfrac{1}{s}$



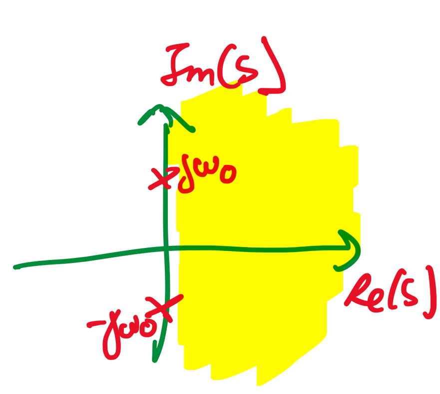
$\Gamma = \{ s \mid Re[s] > Re[s_0] = 0 \}$

ES2 TROVARE $X(s)$ PER $x(t) = \cos(\omega_0 t) 1(t)$



$x(t) = \frac{1}{2} e^{\overset{s_0}{j\omega_0} t} 1(t) + \frac{1}{2} e^{\overset{s_0}{-j\omega_0} t} 1(t)$

$X(s) = \dfrac{1}{2} \dfrac{1}{s - j\omega_0} + \dfrac{1}{2} \dfrac{1}{s + j\omega_0}$



$Re[s] > 0$

$= \dfrac{s + \cancel{j\omega_0} + s - \cancel{j\omega_0}}{2(s - j\omega_0)(s + j\omega_0)} = \dfrac{s}{s^2 - (j\omega_0)^2}$

$= \dfrac{s}{s^2 + \omega_0^2}$

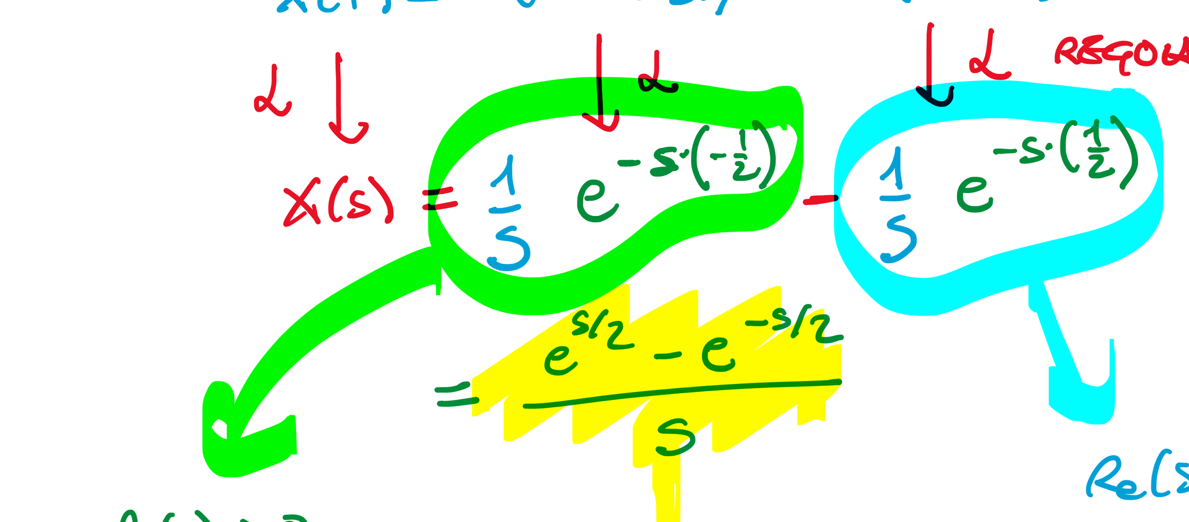
X CASA PROVARE CON $\sin(\omega_0 t) 1(t) \xrightarrow{\mathcal{L}} \dfrac{\omega_0}{s^2 + \omega_0^2}$

ES3 TROVARE $X(s)$ PER $x(t) = rect(t)$

$X(s) = \int_{\cancel{-\infty}\ -\frac{1}{2}}^{\cancel{+\infty}\ 1/2} \cancel{rect(t)}\, e^{-st}\, dt = \left[\dfrac{e^{-st}}{-s} \right]_{-\frac{1}{2}}^{1/2}$

$= \begin{cases} \dfrac{e^{-s/2} - e^{s/2}}{-s} = \dfrac{e^{s/2} - e^{-s/2}}{s} & s \neq 0 \\ \int_{-1/2}^{1/2} 1\, dt = 1 & s = 0 \end{cases}$

$\Gamma = \mathbb{C}$



$x(t) = 1(t - (-\tfrac{1}{2})) - 1(t - \tfrac{1}{2})$

REGOLA DI TRASLAZIONE

$X(s) = \dfrac{1}{s} e^{-s \cdot (-\frac{1}{2})} - \dfrac{1}{s} e^{-s \cdot (\frac{1}{2})}$

$= \dfrac{e^{s/2} - e^{-s/2}}{s}$

$Re(s) > 0$ $Re(s) > 0$

se $\mathbb{C}$

ES4 TROVARE $X(s)$ PER $x(t) = \delta(t)$

$X(s) = \int_{-\infty}^{+\infty} \delta(t) e^{-st}\, dt = e^{-st} \Big|_{t=0} = 1$

$\Gamma = \mathbb{C}$

ES5 TROVARE $X(s)$ PER $x(t) = \delta(t - t_0)$

REGOLA DI TRASLAZIONE

$\delta(t - t_0) \xrightarrow{\mathcal{L}} 1 \cdot e^{-st_0} = e^{-st_0}$

$\Gamma = \mathbb{C}$

ES6 TROVARE $X(s)$ PER $x(t) = \delta'(t)$

REGOLA DI DERIVAZIONE

$\delta'(t) \xrightarrow{\mathcal{L}} 1 \cdot s = s$ $\Gamma = \mathbb{C}$

NOTA $\delta^{(k)}(t) \xrightarrow{\mathcal{L}} s^k$

DERIVATA K-ESIMA

ES7 TROVARE $X_k(s)$ PER $x_k(t) = t^k e^{s_0 t} 1(t)$

K INTERO, ≥ 0

$K=0$ $x_0(t) = e^{s_0 t} 1(t) \xrightarrow{\mathcal{L}} X_0(s) = \dfrac{1}{s - s_0}$

$K=1$ $x_1(t) = t\, x_0(t) \xrightarrow[\text{REGOLA DI MOLT. PER } t]{\mathcal{L}} X_1(s) = -X_0'(s)$

$= +1 \cdot +1 \cdot \dfrac{1}{(s-s_0)^2}$

$= \dfrac{1}{(s-s_0)^2}$

$K=2$ $x_2(t) = t\, x_1(t) \xrightarrow{\mathcal{L}} X_2(s) = -X_1'(s)$

$= +1 \cdot +2 \cdot \dfrac{1}{(s-s_0)^3}$

$= \dfrac{2}{(s-s_0)^3}$

$K=3$ $x_3(t) = t\, x_2(t) \xrightarrow{\mathcal{L}} X_3(s) = -X_2'(s)$

$= +2 \cdot +3 \cdot \dfrac{1}{(s-s_0)^4}$

$= \dfrac{2 \cdot 3}{(s-s_0)^4}$

$x_k(t) = t^k e^{s_0 t} 1(t) \xrightarrow{\mathcal{L}} \dfrac{k!}{(s-s_0)^{k+1}} = X_k(s)$

X INDUZIONE

$x_{k+1}(t) = t\, x_k(t) \xrightarrow{\mathcal{L}} X_{k+1}(s) = -X_k'(s)$

$= +k! \cdot +(k+1) \cdot \dfrac{1}{(s-s_0)^{k+2}}$

$= \dfrac{(k+1)!}{(s-s_0)^{k+2}}$ ✓ DIMOSTRATO!

NOTAZIONE STANDARD

$\boxed{\dfrac{t^k}{k!} e^{s_0 t} 1(t) \xrightarrow{\mathcal{L}} \dfrac{1}{(s-s_0)^{k+1}}}$

NOTA $y_k(t) = \dfrac{t^k}{k!} 1(t) = \underbrace{1 * 1 * \ldots * 1(t)}_{k+1} \xrightarrow{\mathcal{L}} \dfrac{1}{s^{k+1}}$

$y_0(t) = 1(t)$

$y_1(t) = t \cdot 1(t) = 1 * 1(t)$

$y_2(t) = \dfrac{t^2}{2} 1(t) = 1 * 1 * 1(t)$

$y_3(t) = \dfrac{t^3}{3!} 1(t) = 1 * 1 * 1 * 1(t)$