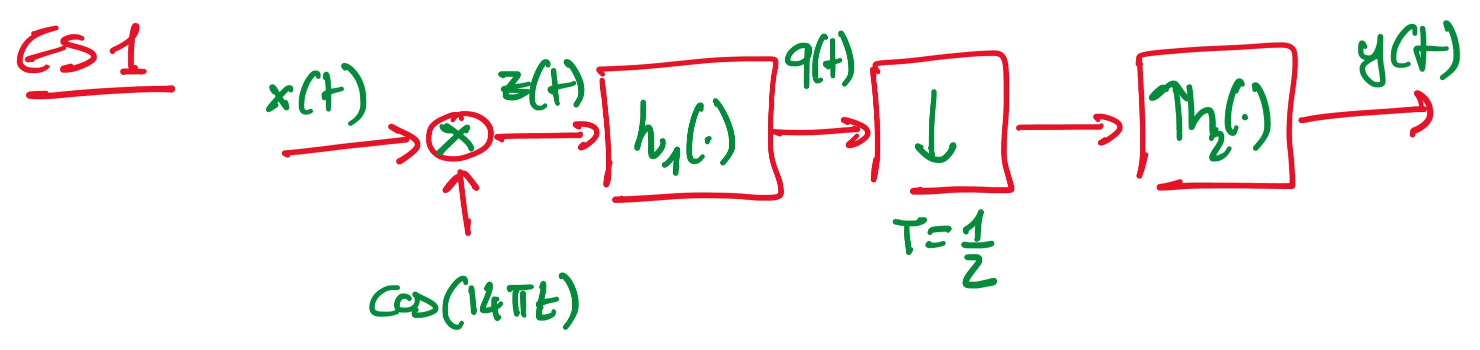
Le24

Friday, 16 May 2025 10:32

ES 1



$$X(j\omega) = rect\left(\frac{\omega}{4\pi}\right)\cdot\left(1 - triang\left(\frac{\omega}{2\pi}\right)\right)$$

$$H_1(j\omega) = 2 - 2\,rect\left(\frac{\omega}{28\pi}\right)$$

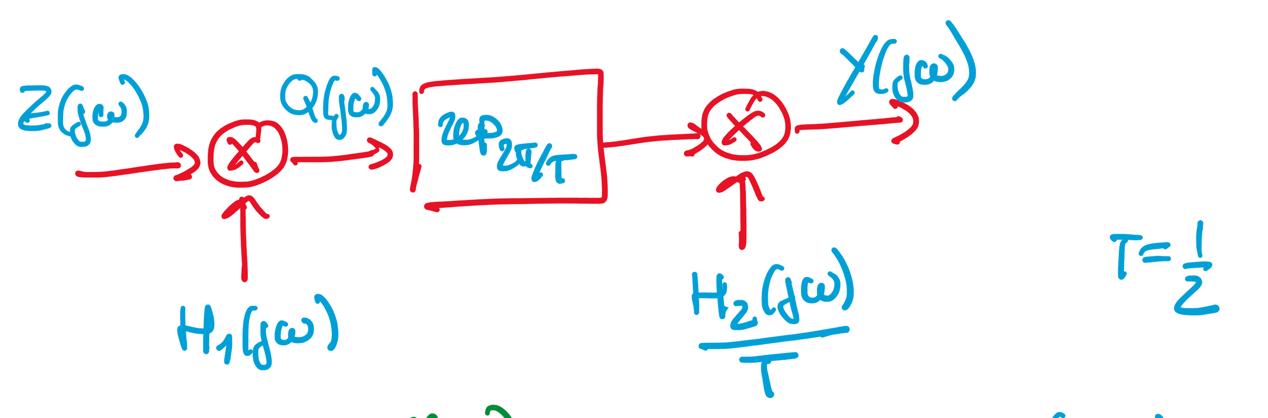
$$H_2(j\omega) = \frac{1}{2}\,rect\left(\frac{\omega}{4\pi}\right)$$

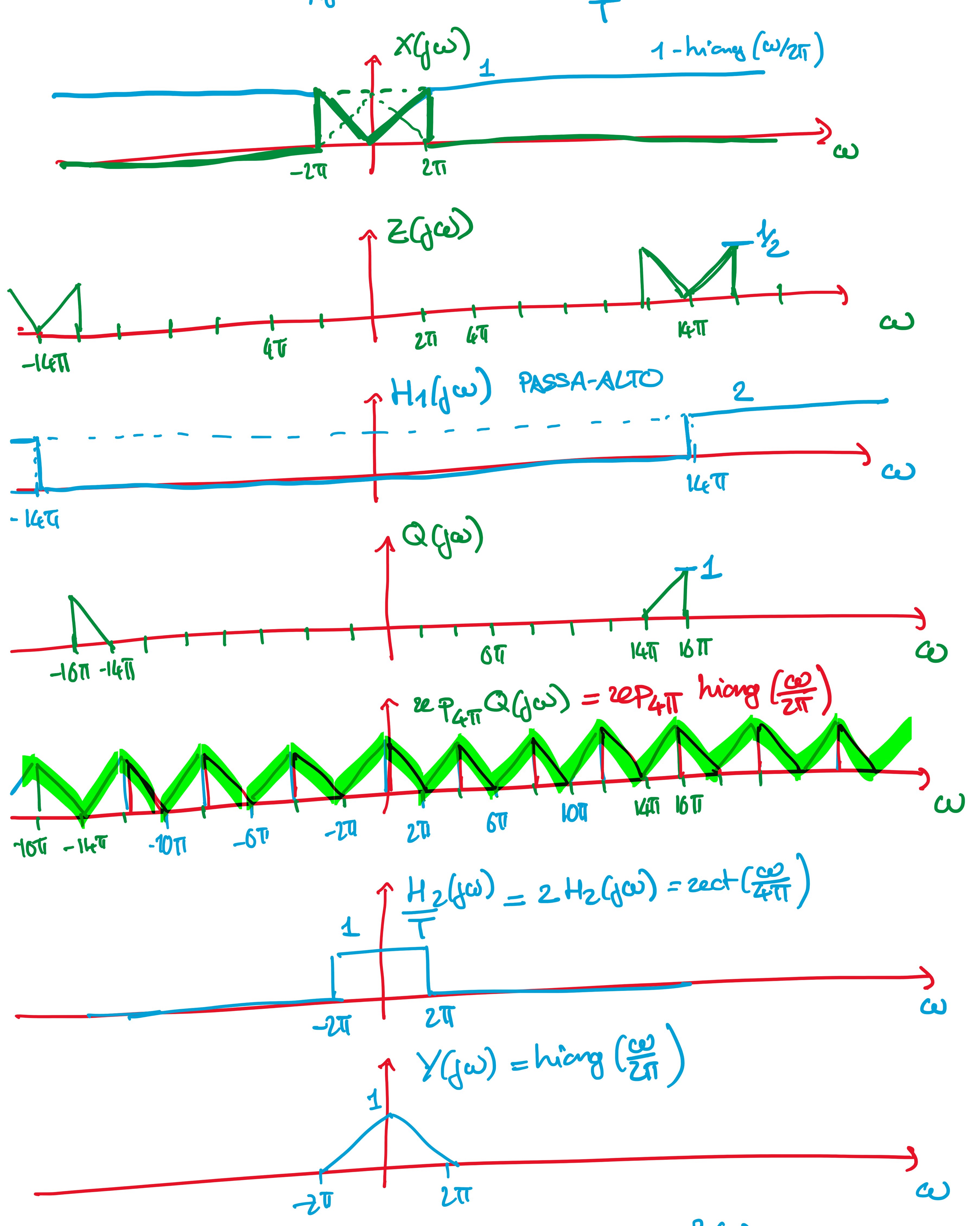
$$y(t) = ?$$

$$z(t) = x(t)\cos(14\pi t) = \frac{1}{2}x(t)e^{j14\pi t} + \frac{1}{2}x(t)e^{-j14\pi t}$$

$\downarrow \mathcal{F}$

$$Z(j\omega) = \frac{1}{2}X(j(\omega-14\pi)) + \frac{1}{2}X(j(\omega+14\pi))$$





$$y(t) = sinc^2(t)$$

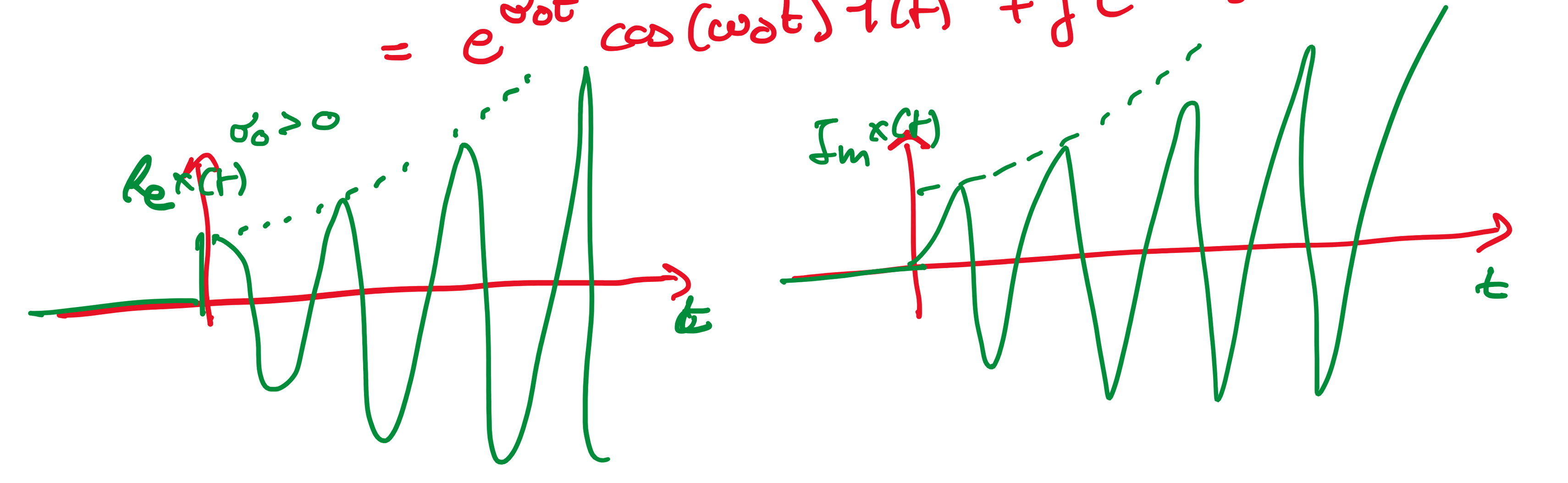
ES 2 CALCOLARE $X(s)$ PER $x(t) = e^{s_0 t}1(t)$

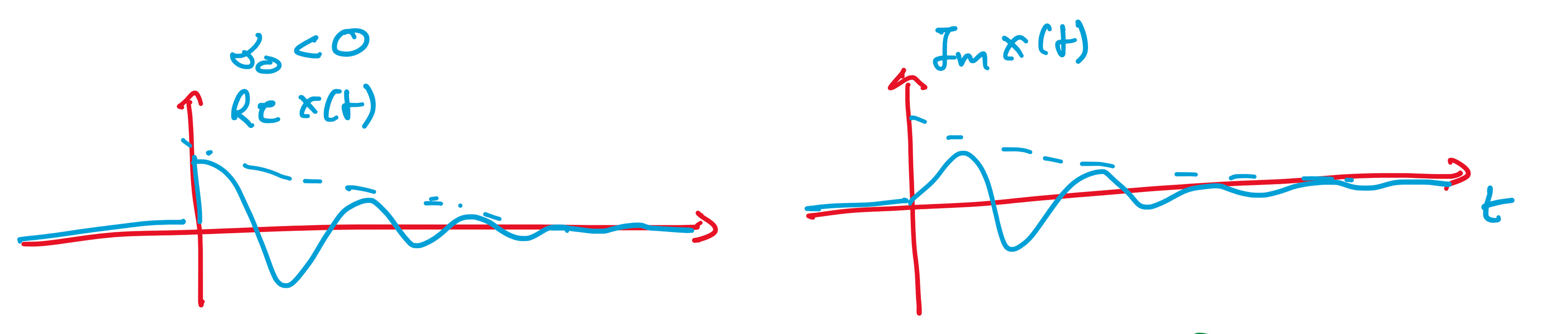
$s_0 \in \mathbb{C}$

$$s_0 = \sigma_0 + j\omega_0$$

$$x(t) = e^{\sigma_0 t}e^{j\omega_0 t}1(t)$$

$$= e^{\sigma_0 t}\cos(\omega_0 t)1(t) + j\,e^{\sigma_0 t}\sin(\omega_0 t)1(t)$$





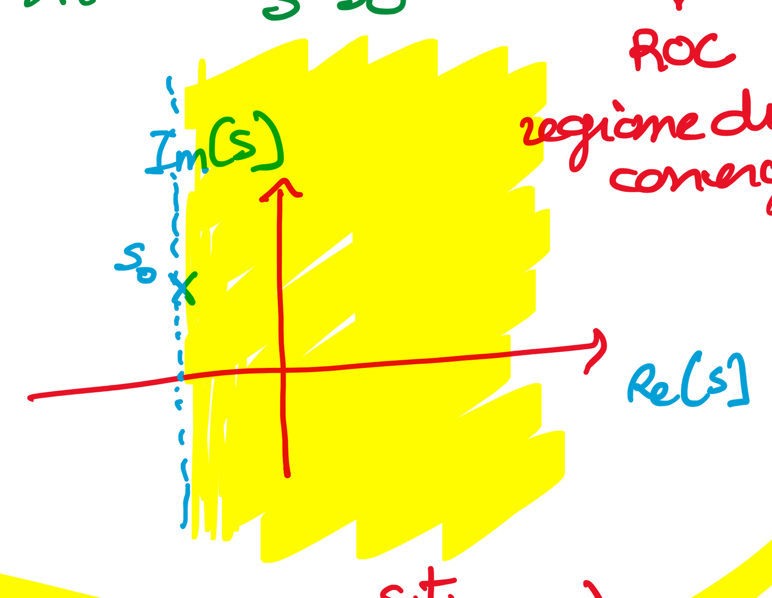
$$X(s) = \int_{\cancel{-\infty}\ 0}^{+\infty} e^{s_0 t}\,\cancel{1(t)}\,e^{-st}\,dt = \int_0^{+\infty} e^{(s_0-s)t}\,dt$$

$$= \left.\frac{e^{(s_0-s)t}}{s_0-s}\right|_0^{+\infty} = \frac{\lim_{t\to+\infty} e^{(s_0-s)t} - 1}{s_0-s}$$

$$= \begin{cases} \dfrac{-1}{s_0-s} = \dfrac{1}{s-s_0} & Re[s_0-s] < 0 \\ \text{l'integrale NON converge} & Re[s_0-s] \geq 0 \end{cases}$$

$$x(t) = e^{s_0 t}1(t) \xrightarrow{\mathcal{L}} X(s) = \frac{1}{s-s_0} \quad \underbrace{Re[s_0] < Re[s]}_{\text{ROC}}$$

regione di convergenza



ES 3 CALCOLARE $X(s)$ PER $x(t) = -e^{s_0 t}1(-t)$

$$X(s) = \int_{-\infty}^{\cancel{+\infty}\ 0} -e^{s_0 t}\,\cancel{1(-t)}\,e^{-st}\,dt$$

$$= -\int_{-\infty}^{0} e^{(s_0-s)t}\,dt = \left.\frac{e^{(s_0-s)t}}{s-s_0}\right|_{-\infty}^{0}$$

$$= \frac{1 - \lim_{t\to-\infty} e^{(s_0-s)t}}{s-s_0}$$

$$= \begin{cases} \dfrac{1}{s-s_0} & Re[s_0-s] > 0 \\ \text{non converge} & Re[s_0-s] \leq 0 \end{cases}$$

$$x(t) = -e^{s_0 t}1(-t) \xrightarrow{\mathcal{L}} X(s) = \frac{1}{s-s_0} \quad Re[s] < Re[s_0]$$