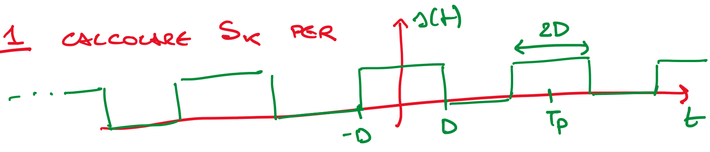
Le19

Thursday, 17 April 2025 08:30

ES 1 CALCOLARE S_K PER



$2D < T_p$

DUTY CYCLE $d = \frac{2D}{T_p} < 1$

$s(t) = 2\Pi_{T_p}\left(\frac{t}{2D}\right)$

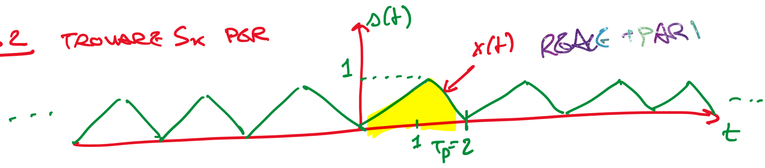
$x(t) = 2\,\mathrm{rect}\left(\frac{t}{2D}\right) \xrightarrow{\mathcal{F}} X(j\omega) = 2D\,\mathrm{sinc}\left(\frac{\omega 2D}{2\pi}\right)$

$S_K = \frac{1}{T_p} X(jK\omega_0)$ $\quad \omega_0 = \frac{2\pi}{T_p}$

$= \frac{1}{T_p} \cdot 2D\,\mathrm{sinc}\left(\frac{2D}{2\pi} \cdot K\omega_0 \cdot \frac{2\pi}{T_p}\right)$

$= \frac{2D}{T_p} \cdot \mathrm{sinc}\left(K \cdot \frac{2D}{T_p}\right) = d\,\mathrm{sinc}(Kd)$

ES 2 TROVARE S_K PER



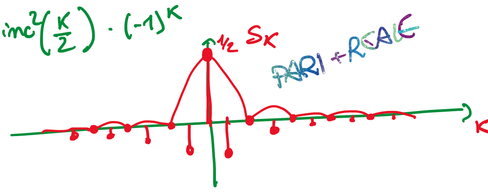
$s(t) = 2\Pi_2\, x(t)$ $\quad x(t) = \mathrm{triangle}(t-1)$

$\xrightarrow{\mathcal{F}}$ $S_K = \frac{1}{T_p} X(jK\omega_0)$ $\quad T_p = 2$, $\omega_0 = \frac{2\pi}{T_p} = \pi$

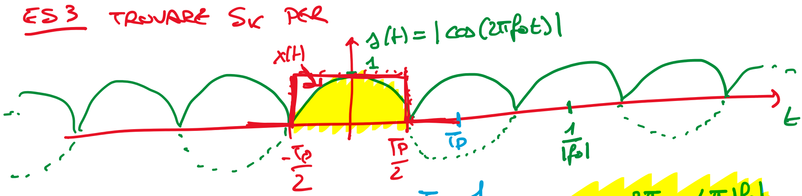
$= \frac{1}{2}\,\mathrm{sinc}^2\left(\frac{K\pi}{2\pi}\right) e^{-jK\pi}$ $\quad \left(e^{-j\pi}\right)^K = (-1)^K$

$= \frac{1}{2}\,\mathrm{sinc}^2\left(\frac{K}{2}\right) \cdot (-1)^K$

$\xrightarrow{\mathcal{F}}$ $X(j\omega) = \mathrm{sinc}^2\left(\frac{\omega}{2\pi}\right) e^{-j\omega \cdot 1}$



ES 3 TROVARE S_K PER



$T_p = \frac{1}{2|f_0|}$ $\quad \omega_0 = \frac{2\pi}{T_p} = 4\pi|f_0|$

$s(t) = 2\Pi_{T_p}\, x(t)$ $\quad x(t) = \cos(2\pi f_0 t) \cdot \mathrm{rect}\left(\frac{t}{T_p}\right)$

$= \frac{1}{2} e^{j2\pi f_0 t}\,\mathrm{rect}\left(\frac{t}{T_p}\right) + \frac{1}{2} e^{-j2\pi f_0 t}\,\mathrm{rect}\left(\frac{t}{T_p}\right)$

$\mathcal{F}$ ↓

$X(j\omega) = \frac{1}{2} \cdot T_p\,\mathrm{sinc}\left(\frac{\omega - 2\pi f_0}{2\pi} T_p\right) + \frac{1}{2} \cdot T_p\,\mathrm{sinc}\left(\frac{(\omega + 2\pi f_0) T_p}{2\pi}\right)$

$= \frac{T_p}{2} \cdot \mathrm{sinc}\left(\frac{\omega - \frac{\omega_0}{2}}{\omega_0}\right) + \frac{T_p}{2}\,\mathrm{sinc}\left(\frac{\omega + \frac{\omega_0}{2}}{\omega_0}\right)$

$= \frac{T_p}{2}\,\mathrm{sinc}\left(\frac{\omega}{\omega_0} - \frac{1}{2}\right) + \frac{T_p}{2}\,\mathrm{sinc}\left(\frac{\omega}{\omega_0} + \frac{1}{2}\right)$

$\mathcal{F}$ ↓

$S_K = \frac{1}{T_p} X(jK\omega_0) = \frac{1}{2}\,\mathrm{sinc}\left(K - \frac{1}{2}\right) + \frac{1}{2}\,\mathrm{sinc}\left(K + \frac{1}{2}\right)$

$= \frac{1}{T_p}\left(\frac{T_p}{2}\,\mathrm{sinc}\left(\frac{K\omega_0}{\omega_0} - \frac{1}{2}\right) + \frac{T_p}{2}\,\mathrm{sinc}\left(\frac{K\omega_0}{\omega_0} + \frac{1}{2}\right)\right)$