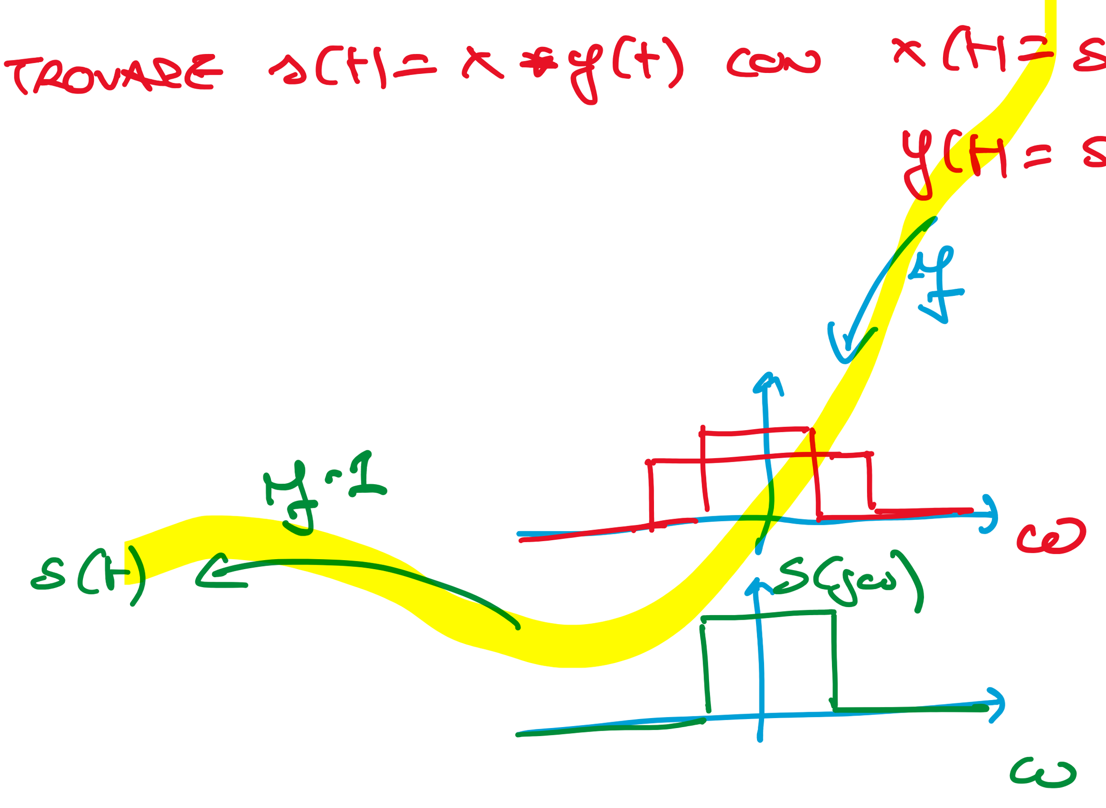
Le18

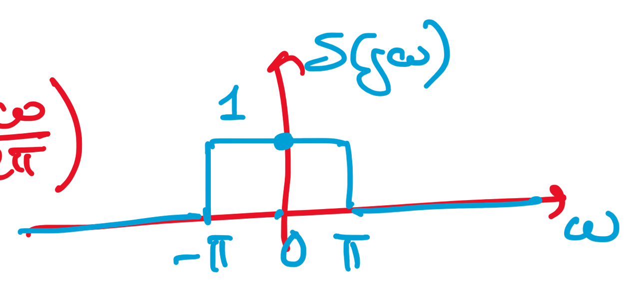
Monday, 14 April 2025 08:13

XCASA TROVARE $s(t) = x * y(t)$ CON $x(t) = \text{sinc}(t/2)$, $y(t) = \text{sinc}(t/3)$



ES1 CALCOLARE A_s E E_s DEL SEGNALE $s(t) = \text{sinc}(t)$

$$s(t) = \text{sinc}(t) \xrightarrow{\mathcal{F}} S(j\omega) = \text{rect}\left(\frac{\omega}{2\pi}\right)$$



$$A_s = S(j0) = 1$$

$$E_s = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |S(j\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \text{rect}\left(\frac{\omega}{2\pi}\right) d\omega$$

TEOREMA DI PARSEVAL

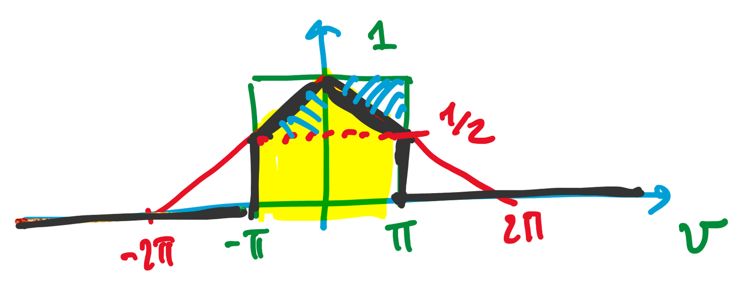
$$= \frac{1}{2\pi}\cdot 2\pi = 1$$

ES2 CALCOLARE AREA DI $s(t) = \text{sinc}^3(t) = \text{sinc}(t)\cdot\text{sinc}^2(t)$

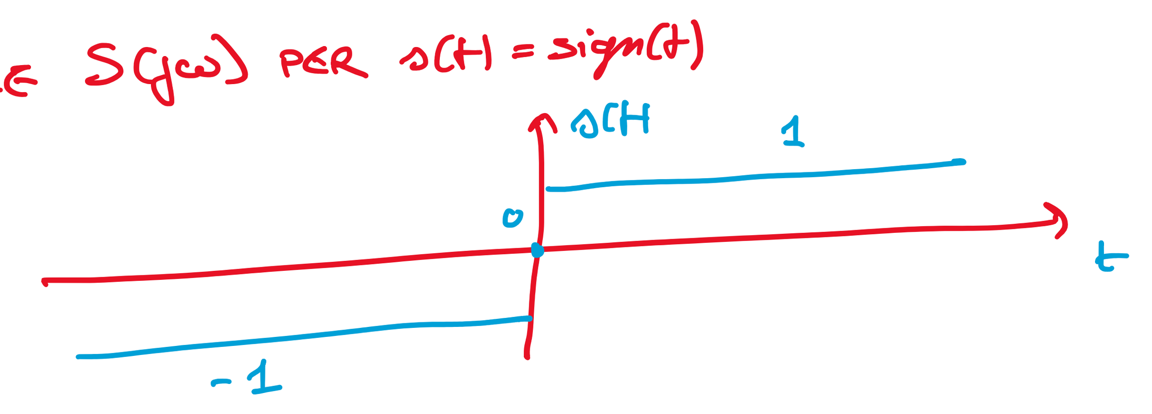
$$S(j\omega) = \frac{1}{2\pi}\,\text{rect}\left(\frac{\omega}{2\pi}\right) * \text{triangle}\left(\frac{\omega}{2\pi}\right)$$

$$S(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \text{rect}\left(\frac{v}{2\pi}\right)\text{triangle}\left(\frac{\omega - v}{2\pi}\right) dv$$

$$A_s = S(j0) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \text{rect}\left(\frac{v}{2\pi}\right)\text{triangle}\left(\frac{+v}{2\pi}\right) dv = \frac{1}{2\pi}\cdot 2\pi\cdot\frac{3}{4} = \frac{3}{4}$$



ES3 CALCOLARE $S(j\omega)$ PER $s(t) = \text{sign}(t)$



$$S(j\omega) = \int_{-\infty}^{+\infty} s(t)\, e^{-j\omega t} dt = -\int_{-\infty}^{0} \cancel{s(t)}\, e^{-j\omega t} dt + \int_{0}^{+\infty} \cancel{s(t)}\, e^{-j\omega t} dt$$

$$= \cancel{\left.\frac{-e^{-j\omega t}}{-j\omega}\right|_{-\infty}^{0}} + \cancel{\left.\frac{e^{-j\omega t}}{-j\omega}\right|_{0}^{+\infty}}$$

indeterminato

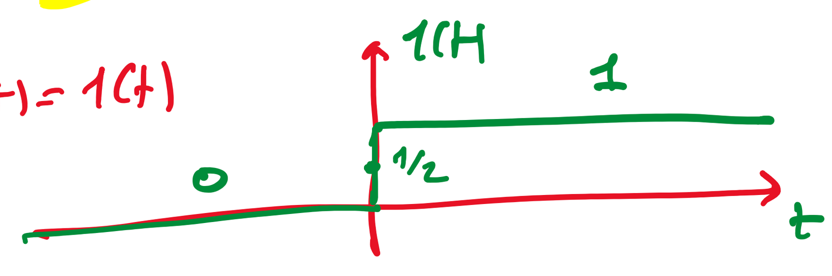
$$y(t) = s'(t) = 2\delta(t)$$

$$Y(j\omega) = j\omega S(j\omega) = 2 \qquad m_s = 0$$

$$S(j\omega) = \frac{Y(j\omega)}{j\omega} + \cancel{m_s 2\pi\delta(\omega)}$$

$$= \frac{2}{j\omega}$$

ES4 TROVARE $S(j\omega)$ PER $s(t) = 1(t)$



$$s(t) = \frac{1}{2}\,\text{sign}(t) + \frac{1}{2}\cdot 1$$

$$S(j\omega) = \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{j\omega} + \frac{1}{\cancel{2}}\cdot\cancel{2}\pi\delta(\omega) = \frac{1}{j\omega} + \pi\delta(\omega)$$

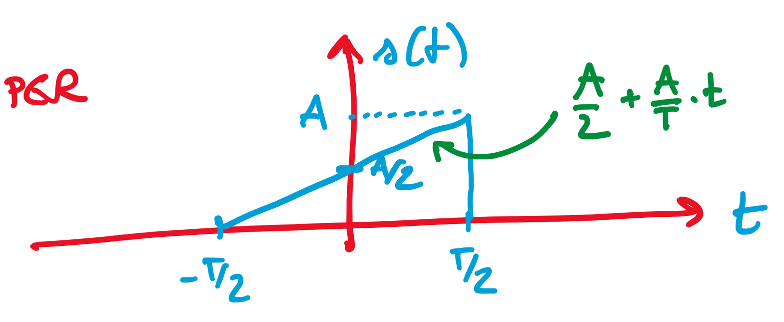
$$y(t) = s'(t) = \delta(t)$$

$$Y(j\omega) = j\omega S(j\omega) = 1 \qquad m_s = \frac{1}{2}$$

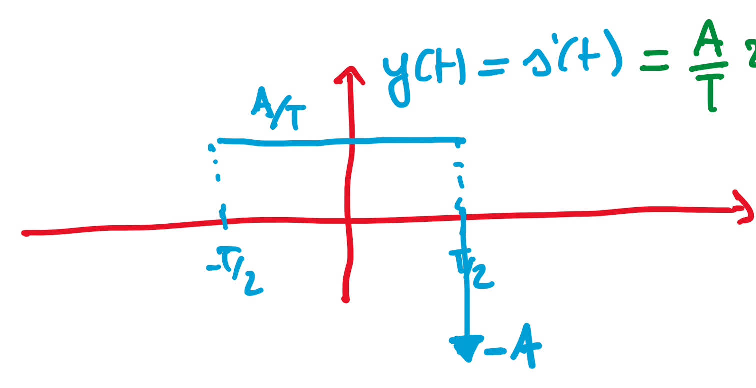
$$S(j\omega) = \frac{Y(j\omega)}{j\omega} + m_s\cdot 2\pi\delta(\omega)$$

$$= \frac{1}{j\omega} + \frac{1}{\cancel{2}}\cdot\cancel{2}\pi\delta(\omega)$$

ES5 TROVARE $S(j\omega)$ PER



$$y(t) = s'(t) = \frac{A}{T}\,\text{rect}\left(\frac{t}{T}\right) - A\delta(t - T/2)$$



REGOLA DI DERIVAZIONE

$$y(t) = s'(t) = \frac{A}{T}\,\text{rect}\left(\frac{t}{T}\right) - A\delta\left(t - T/2\right)$$

$$Y(j\omega) = j\omega S(j\omega) = \frac{A}{\cancel{T}}\cancel{T}\,\text{sinc}\left(\frac{\omega T}{2\pi}\right) - A\cdot e^{-j\omega\cdot T/2} \qquad m_s = 0$$

simmetria hermitiana

$$S(j\omega) = \frac{Y(j\omega)}{j\omega} = \frac{A\,\text{sinc}\left(\frac{\omega T}{2\pi}\right) - A e^{-j\omega T/2}}{j\omega}$$

REGOLA DI MOLTIPLICAZIONE PER t

$$s(t) = \left(\frac{A}{2} + \frac{A}{T}\cdot t\right)\cdot\text{rect}\left(\frac{t}{T}\right)$$

$$= \frac{A}{2}\,\text{rect}\left(\frac{t}{T}\right) + \frac{A}{T}\cdot t\,\text{rect}\left(\frac{t}{T}\right)$$

$$S(j\omega) = \frac{A}{2}\cdot T\,\text{sinc}\left(\frac{\omega T}{2\pi}\right) + \frac{A}{\cancel{T}}\cdot j\frac{d}{d\omega}\left(\cancel{T}\,\text{sinc}\left(\frac{\omega T}{2\pi}\right)\right)$$

$$= \underbrace{\frac{AT}{2}\cdot\text{sinc}\left(\frac{\omega T}{2\pi}\right)} + j\frac{AT}{2\pi}\,\text{sinc}'\left(\frac{\omega T}{2\pi}\right)$$

$$\frac{A\cancel{T}}{\cancel{2}}\,\frac{\sin\left(\cancel{\pi}\cdot\frac{\omega T}{\cancel{2\pi}}\right)}{\cancel{\pi}\cdot\frac{\omega \cancel{T}}{\cancel{2\pi}}} = \frac{A\sin(\omega T/2)}{\omega}$$