

ES1 TROVARE $S(f\omega)$ PER $s(t) = \delta(t-t_0)$

$$S(f\omega) = \int_{-\infty}^{+\infty} \delta(t-t_0) e^{-j\omega t} dt = e^{-j\omega t_0}$$

PROPRIETÀ DI TRASLAZIONE

$$\delta(t) \xrightarrow{f} 1$$

$$\delta(t-t_0) \xrightarrow{f} 1 \cdot e^{-j\omega t_0}$$

ES2 TROVARE $S(f\omega)$ PER $s(t) = e^{j\omega_1 t}$

PROPRIETÀ DI MODULAZIONE $s(t) = 1 \cdot e^{j\omega_1 t}$

$$1 \xrightarrow{f} 2\pi \delta(\omega)$$

$$1 \cdot e^{j\omega_1 t} \xrightarrow{f} 2\pi \delta(\omega - \omega_1) = S(f\omega)$$

$$\frac{1}{2\pi} \int_{-\infty}^{+\infty} S(f\omega) e^{j\omega t} d\omega = \int_{-\infty}^{+\infty} \delta(\omega - \omega_1) e^{j\omega t} d\omega = e^{j\omega_1 t} = s(t)$$

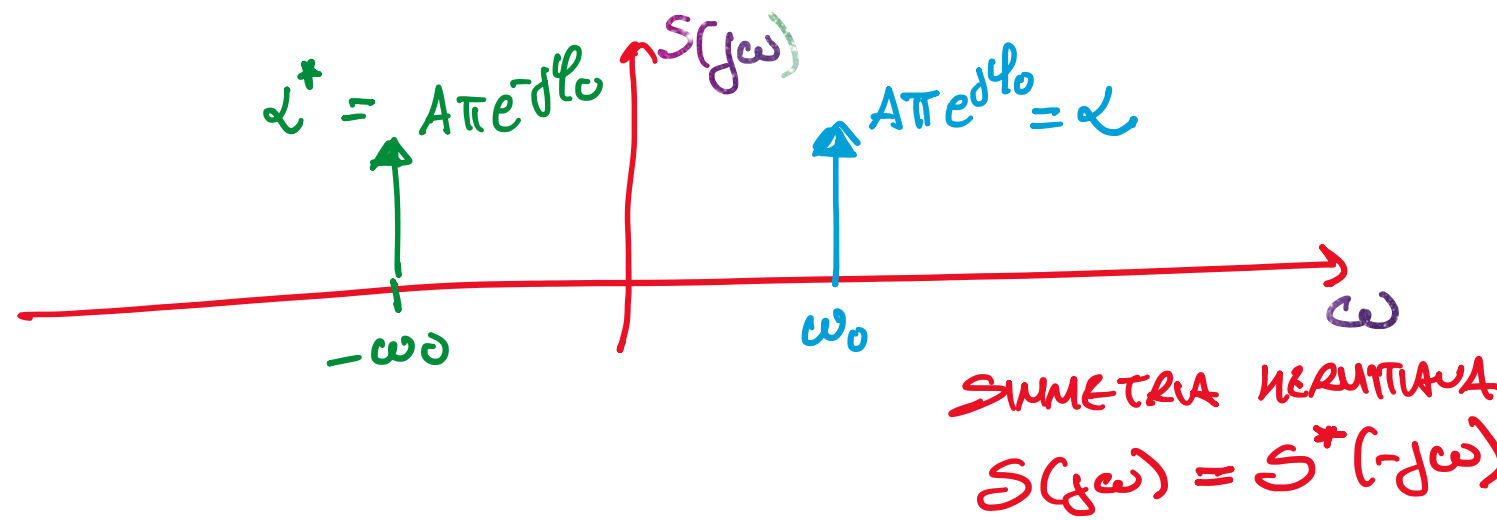
$$\delta(t-t_0) \xrightarrow{f} e^{-j\omega t_0}$$

$$e^{j\omega_1 t} \xrightarrow{f} 2\pi \delta(\omega - \omega_1)$$

ES3 TROVARE $S(f\omega)$ PER $s(t) = \frac{A}{2} \cos(\omega_0 t + \varphi_0)$

$$s(t) = \frac{A}{2} e^{j\varphi_0} e^{j\omega_0 t} + \frac{A}{2} e^{-j\varphi_0} e^{-j\omega_0 t}$$

$$S(f\omega) = \frac{A}{2} e^{j\varphi_0} 2\pi \delta(\omega - \omega_0) + \frac{A}{2} e^{-j\varphi_0} 2\pi \delta(\omega + \omega_0)$$



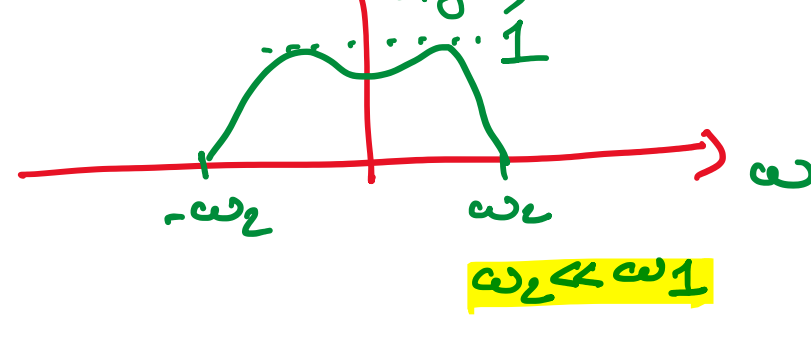
SIMMETRIA HERMITIANA

$$S(f\omega) = S^*(-j\omega)$$

$$S(-j\omega) = S^*(j\omega)$$

(il segnale $s(t)$ è reale)

ES4 TROVARE $S(f\omega)$ PER $s(t) = x(t) \cos(\omega_1 t)$ con

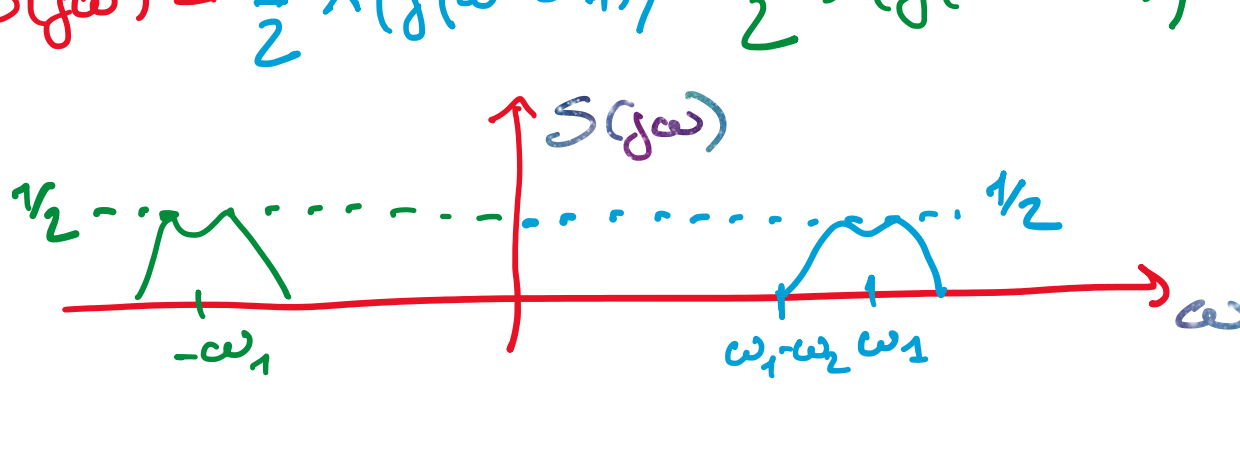


$$\omega_2 \ll \omega_1$$

REGOLA DI MODULAZIONE

$$s(t) = x(t) \cdot \left(\frac{1}{2} e^{j\omega_1 t} + \frac{1}{2} e^{-j\omega_1 t} \right)$$

$$S(f\omega) = \frac{1}{2} X(f(\omega - \omega_1)) + \frac{1}{2} X(f(\omega + \omega_1))$$



REGOLA DEL PRODOTTO

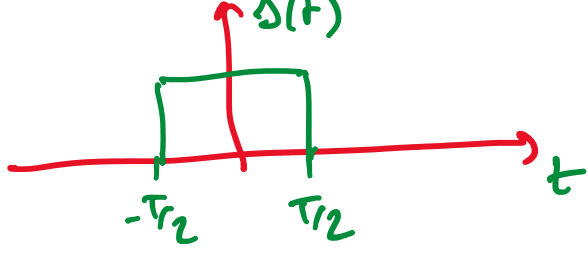
$$s(t) = x(t) \cos(\omega_1 t)$$

$$S(f\omega) = \frac{1}{2} [X(f\omega) * \delta(\omega - \omega_1) + X(f\omega) * \delta(\omega + \omega_1)]$$

$$= \frac{1}{2} X(f\omega) * \delta(\omega - \omega_1) + \frac{1}{2} X(f\omega) * \delta(\omega + \omega_1)$$

$$= \frac{1}{2} X(f(\omega - \omega_1)) + \frac{1}{2} X(f(\omega + \omega_1))$$

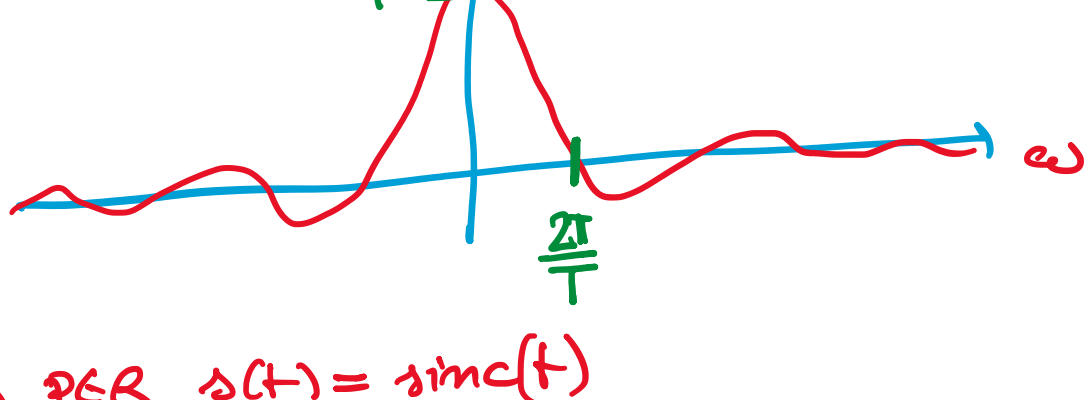
ES5 TROVARE $S(f\omega)$ PER $s(t) = \text{rect}(t/T)$



REGOLA DI SCALET

$$\text{rect}(t) \xrightarrow{f} \text{sinc}\left(\frac{\omega}{2\pi}\right)$$

$$\text{rect}\left(\frac{t}{T}\right) \xrightarrow{f} T \cdot \text{sinc}\left(\frac{\omega \cdot T}{2\pi}\right) = T \text{sinc}\left(\frac{\omega}{2\pi/T}\right)$$



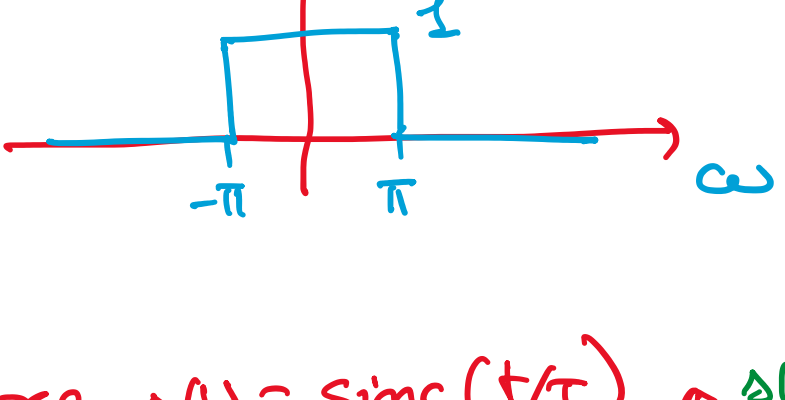
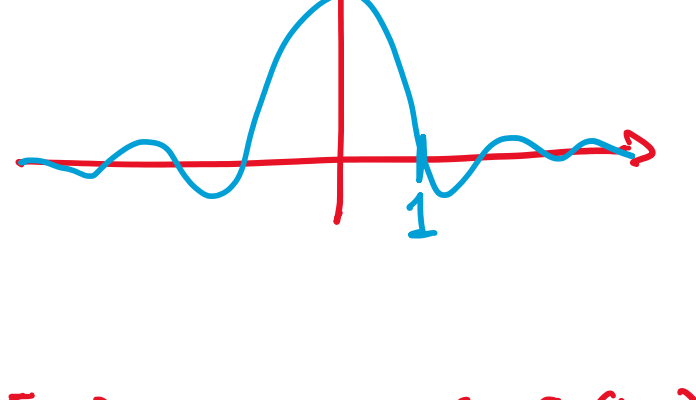
ES6 TROVARE $S(f\omega)$ PER $s(t) = \text{sinc}(t)$

$$x(t) = \text{rect}(t) \xrightarrow{f} X(f\omega) = \text{sinc}\left(\frac{\omega}{2\pi}\right)$$

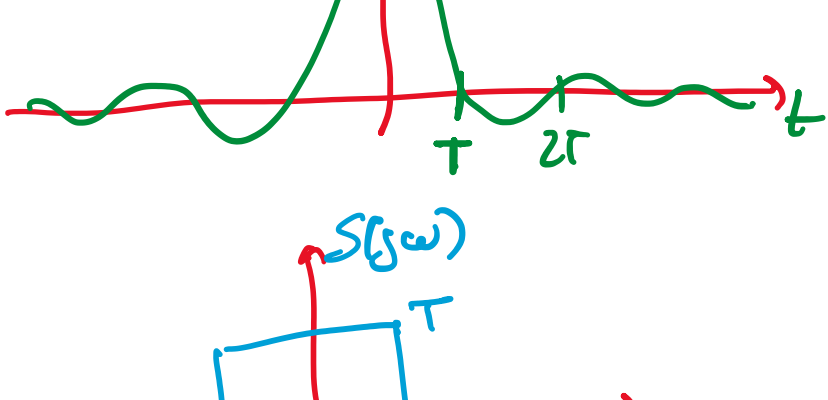
$$y(t) = X(f\omega) = \text{sinc}\left(\frac{t}{2\pi}\right) \xrightarrow{f} Y(f\omega) = 2\pi \text{rect}(\omega) = 2\pi \text{rect}\left(\frac{\omega}{2\pi}\right)$$

$$z(t) = y\left(\frac{t}{2\pi}\right) = y(t \cdot 2\pi) = \text{sinc}\left(\frac{t \cdot 2\pi}{2\pi}\right) = \text{sinc}(t) \xrightarrow{f} \frac{1}{2\pi} Y(f\omega \cdot \frac{1}{2\pi})$$

$$= \frac{1}{2\pi} \cdot 2\pi \text{rect}\left(\frac{\omega}{2\pi}\right) = \text{rect}\left(\frac{\omega}{2\pi}\right)$$

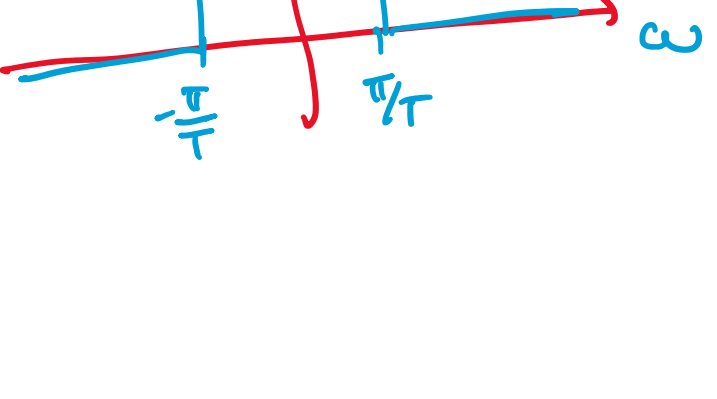


ES7 CALCOLORE $S(f\omega)$ PER $s(t) = \text{sinc}(t/T)$



$$\text{sinc}(t) \xrightarrow{f} \text{rect}\left(\frac{\omega}{2\pi}\right)$$

$$\text{sinc}\left(\frac{t}{T}\right) \xrightarrow{f} T \cdot \text{rect}\left(\frac{\omega}{2\pi/T}\right)$$



ES8 CALCOLORE $S(f\omega)$ PER $s(t) = \text{triangle}(t/T)$

$$s(t) = \text{rect} * \text{rect}(t/T)$$

REGOLA DI CONVOLUZIONE

$$x(t) = \text{rect} * \text{rect}(t)$$

$$X(f\omega) = \text{sinc}\left(\frac{\omega}{2\pi}\right) \cdot \text{sinc}\left(\frac{\omega}{2\pi}\right) = \text{sinc}^2\left(\frac{\omega}{2\pi}\right)$$

$$\text{triangle}(t) \xrightarrow{f} \text{sinc}^2\left(\frac{\omega}{2\pi}\right)$$

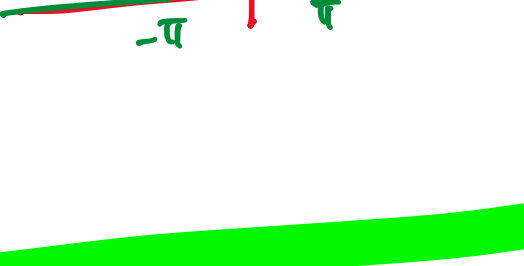
REGOLA DI SCALET

$$s(t) = \text{triangle}(t/T) \xrightarrow{f} T \cdot \text{sinc}^2\left(\frac{\omega}{2\pi/T}\right)$$

ES9 CALCOLORE

$$x(t) = \text{sinc} * \text{sinc}(t)$$

$$X(f\omega) = \text{rect}\left(\frac{\omega}{2\pi}\right) \cdot \text{rect}\left(\frac{\omega}{2\pi}\right) = \text{rect}\left(\frac{\omega}{2\pi}\right)$$



$$x(t) = \text{sinc}(t)$$

$$\text{sinc} * \text{sinc}(t) = \text{sinc}(t)$$