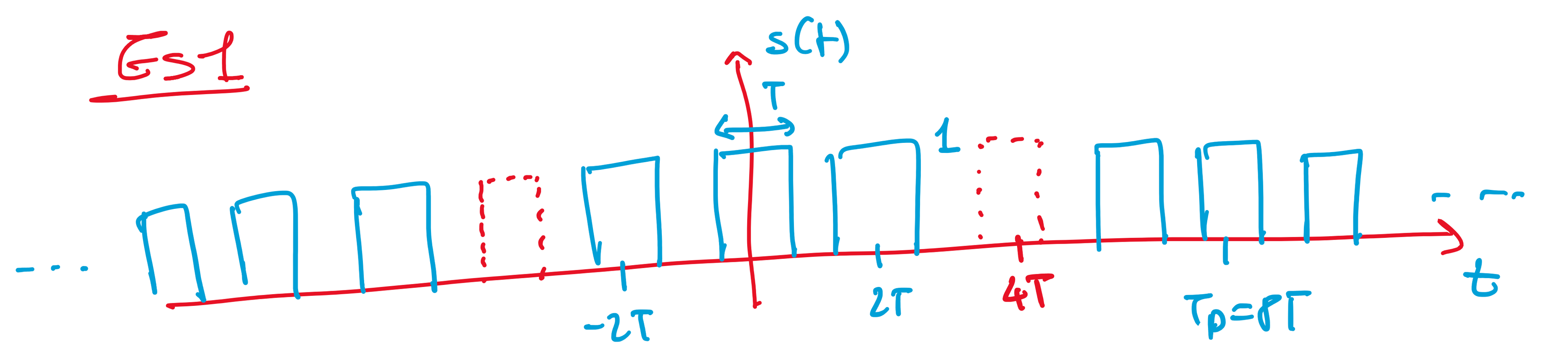
Le16

Thursday, 10 April 2025 08:34

Es1



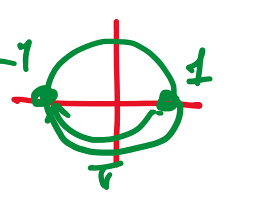
$$s(t) = \underbrace{rep_{2T}\, rect\left(\frac{t}{T}\right)}_{\substack{\text{onda quadra} \\ d=1/2 \\ \text{PERIODO } 2T}} - \underbrace{rep_{8T}\, rect\left(\frac{t-4T}{T}\right)}_{\substack{\text{onda quadra} \\ d=1/8 \\ \text{PERIODO } 8T}}$$

PERIODO 8T

S_K $\qquad \omega_0 = \frac{\pi}{4T}$

$X_m = \frac{1}{2}\,\mathrm{sinc}\left(\frac{m}{2}\right)$ $\qquad \omega_0' = \frac{2\pi}{2T} = \pi/T$

$Y_K = \frac{1}{8}\,\mathrm{sinc}\left(\frac{K}{8}\right) \cdot \underbrace{e^{-jK\overset{\pi}{\cancel{\omega_0 4T}}}}_{(e^{-j\pi})^K = (-1)^K}$ $\qquad \omega_0 = \frac{2\pi}{8T} = \frac{\pi}{4T}$

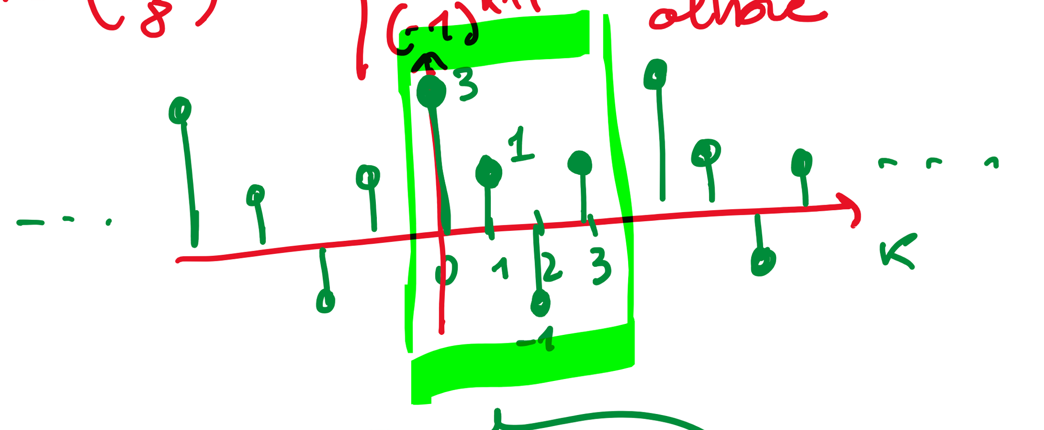


$$S_K = \begin{cases} X_m & K=4m \\ 0 & \text{altrove} \end{cases} - Y_K$$

$$= \begin{cases} \frac{1}{2}\,\mathrm{sinc}\left(\frac{m}{2}\right) \leftarrow K/8 & K=4m \leftarrow m=\frac{K}{4} \\ 0 & \text{altrove} \end{cases} - \frac{1}{8}\,\mathrm{sinc}\left(\frac{K}{8}\right)(-1)^K$$

$$= \begin{cases} \underbrace{\frac{1}{2}\,\mathrm{sinc}\left(\frac{K}{8}\right) - \frac{1}{8}\,\mathrm{sinc}\left(\frac{K}{8}\right)}_{\frac{3}{8}\mathrm{sinc}(K/8)}\cancel{(-1)^K} & K=4m \\ +\frac{1}{8}\,\mathrm{sinc}\left(\frac{K}{8}\right)(-1)^{K+1} & \text{altrove} \end{cases}$$

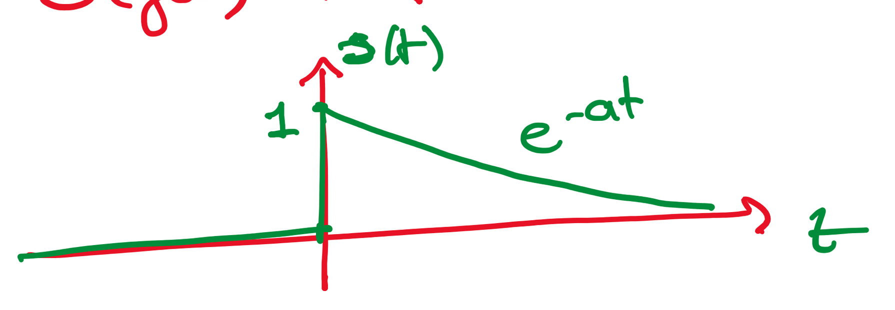
$$= \frac{1}{8}\,\mathrm{sinc}\left(\frac{K}{8}\right) \cdot \begin{cases} 3 & K=4m \\ (-1)^{K+1} & \text{altrove} \end{cases}$$



$$\overset{?}{=} \frac{1}{8}\,\mathrm{sinc}\left(\frac{K}{8}\right) \cdot \left(1 + 2\cos\left(\frac{\pi}{2}K\right)\right)$$

Es2

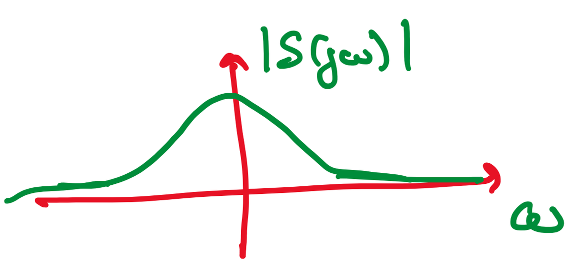
TROVARE $S(j\omega)$ PER $s(t) = e^{-at}\,1(t)$, $a>0$.



$$S(j\omega) = \int_{\cancel{-\infty}\ 0}^{+\infty} \cancel{1(t)}\, e^{-at}\, e^{-j\omega t}\, dt$$

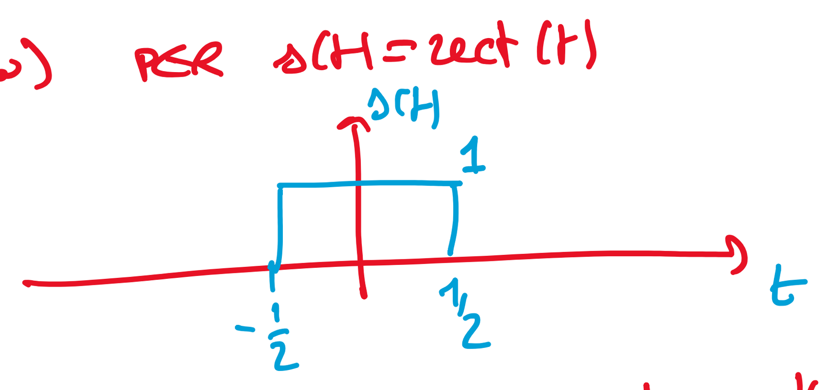
$$= \int_0^{+\infty} e^{-(a+j\omega)t}\, dt = \left.\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right|_0^{+\infty}$$

$$= \frac{0-1}{-(a+j\omega)} = \frac{1}{a+j\omega}$$



Es 3

TROVARE $S(j\omega)$ PER $s(t) = rect(t)$

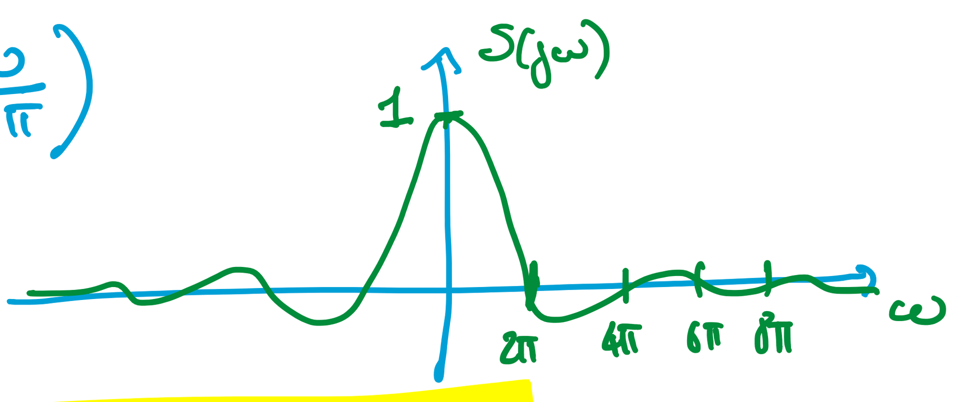


$$S(j\omega) = \int_{\cancel{-\infty}\ -\frac{1}{2}}^{\cancel{+\infty}\ 1/2} \cancel{s(t)}\, e^{-j\omega t}\, dt = \begin{cases} \left.\frac{e^{-j\omega t}}{-j\omega}\right|_{-\frac{1}{2}}^{1/2} & \omega \neq 0 \\ \int_{-\frac{1}{2}}^{1/2} 1\, dt = 1 & \omega = 0 \end{cases}$$

$$= \begin{cases} 1 & \omega = 0 \\ \frac{-e^{-j\omega/2} + e^{j\omega/2}}{+j\omega} & \omega \neq 0 \end{cases}$$

$$\rightarrow \frac{\cancel{2j}\,\sin\left(\frac{\omega}{2\pi}\cdot\pi\right)}{\cancel{j}\,\frac{\omega}{2\pi}\cdot\pi} = \mathrm{sinc}\left(\frac{\omega}{2\pi}\right)$$

$$= \mathrm{sinc}\left(\frac{\omega}{2\pi}\right)$$



$$\boxed{rect(t) \xrightarrow{\mathcal{F}} \mathrm{sinc}\left(\frac{\omega}{2\pi}\right)}$$

Es4

$s(t) = \delta(t)$

$$S(j\omega) = \int_{-\infty}^{+\infty} \delta(t)\, e^{-j\omega t}\, dt = \left. e^{-j\omega t}\right|_{t=0} = e^{-j\omega \cdot 0} = e^0 = 1$$

$$\boxed{\delta(t) \xrightarrow{\mathcal{F}} 1}$$

PROVIAMO AD USARE LA REGOLA DI SIMMETRIA

$$x(t) = \delta(t) \xrightarrow{\mathcal{F}} X(j\omega) = 1$$

$$y(t) = X(jt) = 1 \xrightarrow{\mathcal{F}} Y(j\omega) = 2\pi\, x(-\omega) = 2\pi\,\delta(-\omega) = 2\pi\,\delta(\omega)$$

$$\boxed{1 \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftarrows}} 2\pi\,\delta(\omega)}$$

$S(j\omega) = 2\pi\,\delta(\omega)$

$$s(t) = \cancel{\frac{1}{2\pi}} \int_{-\infty}^{+\infty} \underset{\cancel{2\pi}\,\delta(\omega)}{\cancel{S(j\omega)}}\, e^{j\omega t}\, d\omega = \int_{-\infty}^{+\infty} \delta(\omega)\, e^{j\omega t}\, d\omega$$

$$= \int_{-\infty}^{+\infty} \delta(u)\, e^{jut}\, du = \left. e^{jut}\right|_{u=0} = 1$$

NOTA $S(j\omega) = \int_{-\infty}^{+\infty} 1 \cdot e^{-j\omega t}\, dt = ?$ COSÌ NON SI RIESCE A RISOLVERE