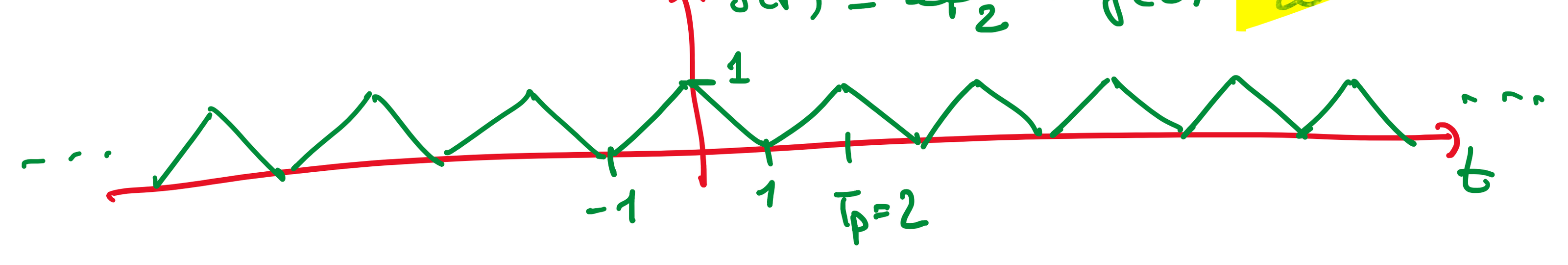
Le15

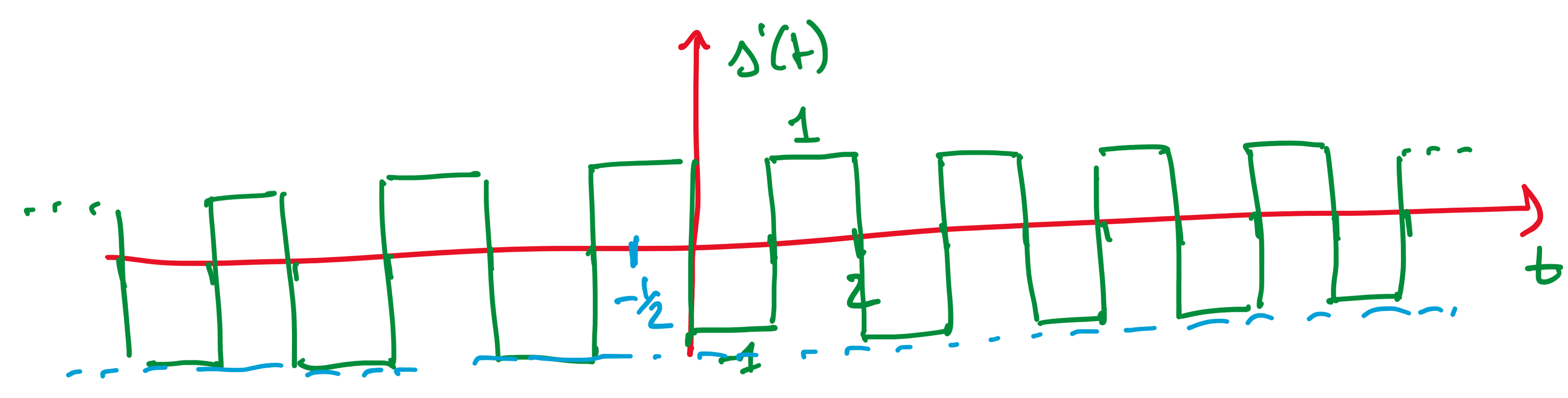
Friday, 4 April 2025 09:18

ES 1 TROVARE S_K PER

$\text{hiong}(t) = \text{rect} * \text{rect}(t)$

$s(t) = \text{rep}_2\, \text{hiong}(t)$ reale e pari





$$y(t) = s'(t) = \underbrace{-1}_{\text{SEGNALE COSTANTE}} + 2\,\underbrace{\text{rep}_2\, \text{rect}\left(t+\tfrac{1}{2}\right)}_{\text{DUTY CYCLE } d=\frac{1}{2},\ \text{TRASLAZIONE } t_0=-\frac{1}{2}}$$ ONDA QUADRA

$$Y_K = -1 \cdot \delta(K) + \cancel{2} \cdot \frac{1}{\cancel{2}} \text{sinc}\left(\frac{K}{2}\right) \cdot e^{+jK\cancel{\omega_0 t_0}\frac{\pi}{2}}$$

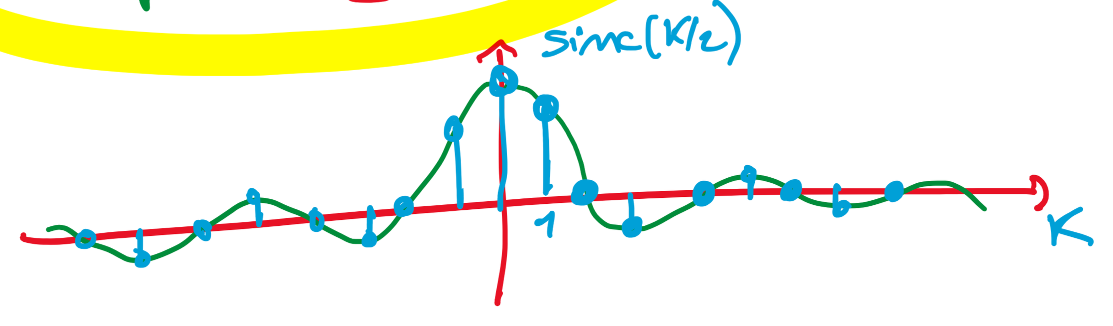
$\omega_0 = 2\pi/2 = \pi$

$\omega_0 t_0 = \pi \cdot -\frac{1}{2} = -\pi/2$

$$Y_K = -\delta(K) + \text{sinc}\left(\frac{K}{2}\right) \underbrace{e^{jK\pi/2}}_{(e^{j\pi/2})^K = j^K}$$

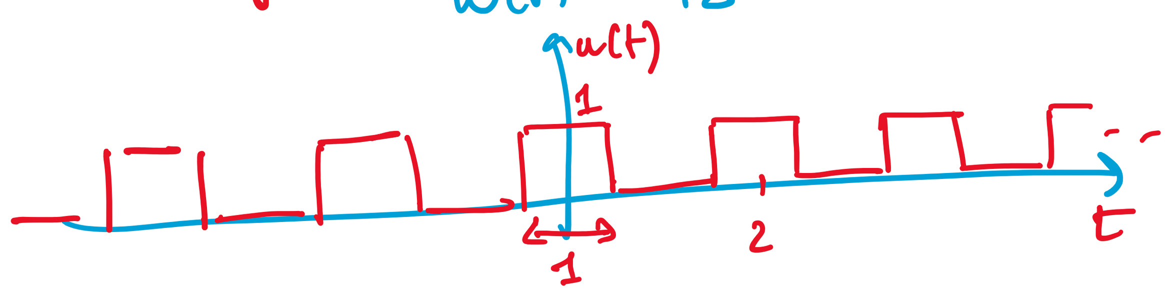
$$S_K = \begin{cases} \dfrac{Y_K}{jK\omega_0} = \dfrac{\text{sinc}\left(\frac{K}{2}\right) \cdot j^K}{jK\omega_0} & K \neq 0 \\ m_s = \dfrac{1}{2} & K = 0 \end{cases}$$

reali e pari



$s(t) = \text{rep}_2\, \underbrace{\text{rect} * \text{rect}(t)}_{\text{hiong}} = u *_{per} u(t)$

$u(t) = \text{rep}_2\, \text{rect}(t)$



$$S_K = T_p \cdot U_K \cdot U_K = 2 \cdot \left(\frac{1}{2}\text{sinc}\left(\frac{K}{2}\right)\right)^2$$

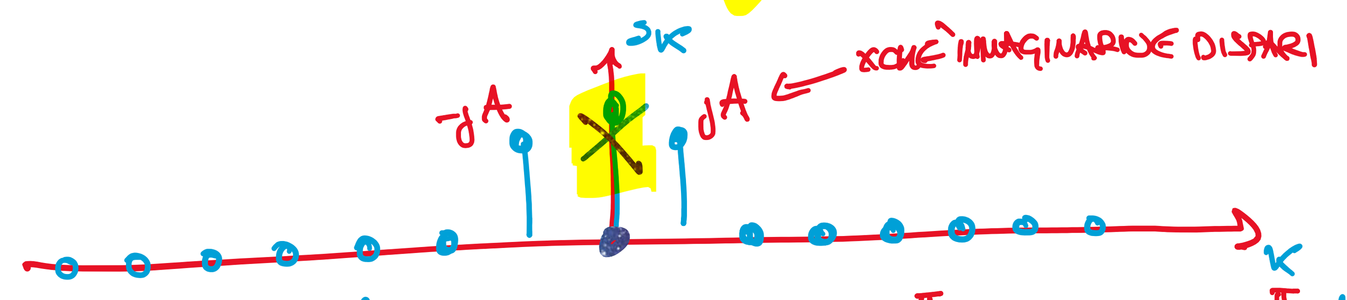
$$= \frac{1}{2}\text{sinc}^2\left(\frac{K}{2}\right)$$

ES2 SIA DATO $s(t)$ TALE CHE SIA

- REALE E DISPARI → media nulla; $\xrightarrow{\mathcal{F}}$ IMMAGINARIO ~~HERMITIANO~~ e DISPARI
- PERIODICO $T_p = 2$
- CON COEFF. DI FOURIER NULLI PER $|K|>1$ OVVERO $S_K = 0$ PER $|K|>1$
- A POTENZA $P_s = 1$

IDENTIFICARE IL SEGNALE $s(t)$.

PERCHÉ IMMAGINARIO E DISPARI



$$s(t) = \sum_{K=-\infty}^{+\infty} S_K e^{jK\omega_0 t} = S_{-1} e^{-j\cancel{\omega_0}\pi t} + \cancel{S_0} + S_1 e^{j\cancel{\omega_0}\pi t}$$

$\omega_0 = \frac{2\pi}{T_p} = \pi$

$$P_s = 1 = \sum_{K=-\infty}^{+\infty} |S_K|^2 = |S_{-1}|^2 + \cancel{|S_0|^2} + |S_1|^2 = A^2 + A^2 = 2A^2$$

$\Downarrow$

$A^2 = \frac{1}{2}$

$A = \pm \frac{1}{\sqrt{2}}$

$$s(t) = -jA\, e^{-j\pi t} + jA\, e^{j\pi t}$$

$$= jA \cdot \left(\frac{e^{j\pi t} - e^{-j\pi t}}{2j}\right) \cdot 2j$$

$$= j \cdot A \cdot \sin(\pi t) \cdot 2j$$

$$= -2A \sin(\pi t)$$

$$s(t) = \mp\sqrt{2}\sin(\pi t)$$

ES3 SIA DATO IL SEGNALE PERIODICO $s(t) = \frac{3}{5}\frac{\sin(\pi t)}{\sin(\frac{\pi}{5}t)}$

SINC PERIODICO PROVARE CON MATLAB!

TROVARE

1) PERIODO T_p
2) S_K
3) P_s, m_s

$$s(t) = \frac{3}{5} \cdot \frac{\frac{e^{j\pi t} - e^{-j\pi t}}{\cancel{2j}}}{\frac{e^{j\frac{\pi}{5}t} - e^{-j\frac{\pi}{5}t}}{\cancel{2j}}} = \frac{3}{5} \cdot \frac{\overset{e^{j\frac{4\pi}{5}t}}{\cancel{e^{j\pi t}}}}{\cancel{e^{j\frac{\pi}{5}t}}} \cdot \frac{1 - e^{-j2\pi t}}{1 - e^{-j\frac{2\pi}{5}t}}$$

$\alpha = e^{-j\frac{2\pi}{5}t}$

$$= \frac{3}{5}\alpha^{-2} \cdot \frac{1-\alpha^5}{1-\alpha}$$ serie geom. troncata

$$= \frac{3}{5}\alpha^{-2}\left(1+\alpha+\alpha^2+\alpha^3+\alpha^4\right)$$

$$= \frac{3}{5}\left(\alpha^{-2}+\alpha^{-1}+1+\alpha+\alpha^2\right)$$

$$s(t) = \frac{3}{5}\sum_{K=-2}^{2}\alpha^K = \frac{3}{5}\sum_{K=-2}^{2} e^{-j\frac{2\pi}{5}Kt}$$

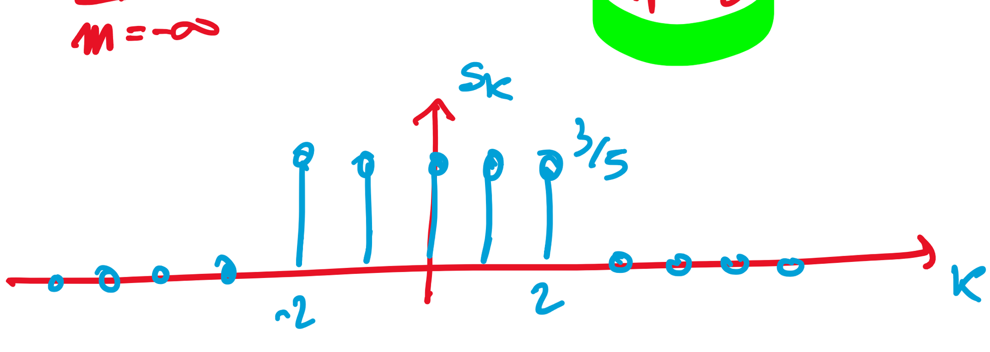
$m = -K \rightarrow$

$$s(t) = \frac{3}{5}\sum_{m=-2}^{2} e^{j\frac{2\pi}{5}mt}$$

$$= \sum_{m=-\infty}^{+\infty} S_m e^{jm\omega_0 t}$$

$\omega_0 = \frac{2\pi}{5} = \frac{2\pi}{T_p}$

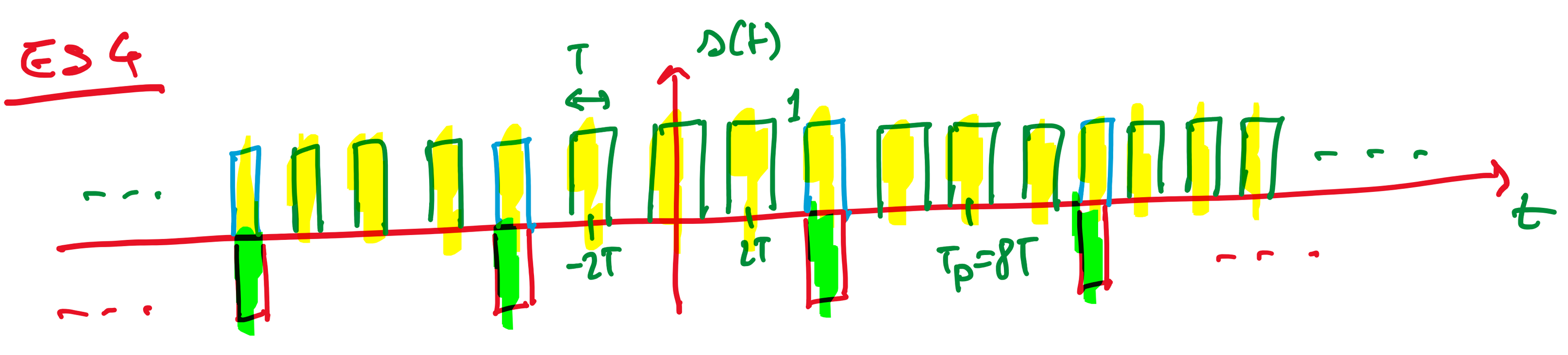
$T_p = 5$



$m_s = S_0 = 3/5$

$$P_s = \sum_{K=-2}^{2}|S_K|^2 = 5 \cdot \left|\frac{3}{5}\right|^2 = \frac{9}{5}$$

ES 4



$$s(t) = \text{rep}_{2T}\,\text{rect}\left(\frac{t}{T}\right) - \text{rep}_{8T}\,\text{rect}\left(\frac{t-4T}{T}\right)$$