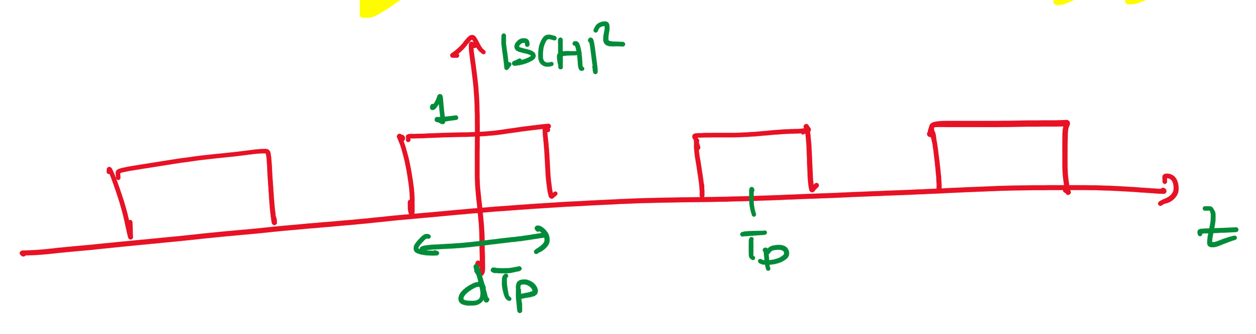
Le14

Wednesday, 2 April 2025 23:04

Es 1

$s(t) = \text{rep}_{T_p}\,\text{rect}\left(\frac{t}{dT_p}\right) \xrightarrow{\mathcal{F}} S_k = d\,\text{sinc}(kd)$

$P_s = E_S = \sum_{k=-\infty}^{+\infty} |S_k|^2 = \sum_{k=-\infty}^{+\infty} d^2\,\text{sinc}^2(kd)$



$P_s = \frac{dT_p}{T_p} = d$

$s(t) = \text{comb}_{T_p}(t) = \text{rep}_{T_p}\,\delta(t) \xrightarrow{\mathcal{F}} S_k = \frac{1}{T_p}$

$P_s = E_S = \sum_{k=-\infty}^{+\infty} |S_k|^2 = \sum_{k=-\infty}^{+\infty} \frac{1}{T_p^2} = \infty$

$s(t) = 1 \xrightarrow{\mathcal{F}} S_k = \delta(k)$

$P_s = E_S = \sum_{k=-\infty}^{+\infty} 1 \cdot \delta(k) = 1$

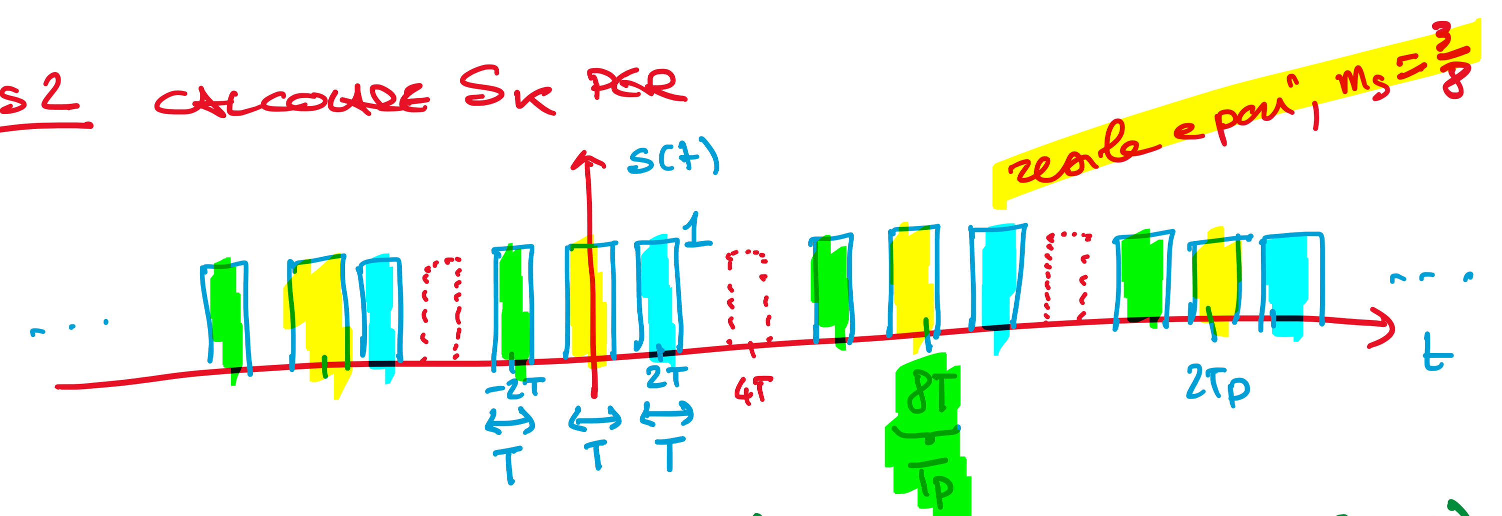
$s(t) = \cos(\omega_0 t + \varphi_0) \xrightarrow{\mathcal{F}} S_k = \underbrace{\frac{1}{2}e^{j\varphi_0}}_{S_1}\delta(k-1) + \underbrace{\frac{1}{2}e^{-j\varphi_0}}_{S_{-1}}\delta(k+1)$

$P_s = E_S = \sum_{k=-\infty}^{+\infty} |S_k|^2 = |S_1|^2 + |S_{-1}|^2$

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

Es 2 CALCOLARE S_k PER

reale e pari, $m_s = \frac{3}{8}$



$s(t) = \underbrace{\text{rep}_{8T}\,\text{rect}\left(\frac{t}{T}\right)}_{u(t)\text{ onda quadra } d=\frac{1}{8}} + \underbrace{\text{rep}_{8T}\,\text{rect}\left(\frac{t+2T}{T}\right)}_{u(t+2T)} + \underbrace{\text{rep}_{8T}\,\text{rect}\left(\frac{t-2T}{T}\right)}_{u(t-2T)}$

$\left(= \text{rep}_{2T}\,\text{rect}\left(\frac{t}{T}\right) - \text{rep}_{8T}\,\text{rect}\left(\frac{t-4T}{8T}\right) \right)$

$u(t) = \text{rep}_{8T}\,\text{rect}\left(\frac{t}{T}\right) \xrightarrow{\mathcal{F}} U_k = \frac{1}{8}\,\text{sinc}\left(\frac{k}{8}\right), \quad \omega_0 = \frac{2\pi}{T_p} = \frac{\pi}{4T}$

$s(t) = u(t) + u(t+2T) + u(t-2T)$

$\downarrow \mathcal{F}$

$S_k = U_k + U_k\, e^{-jk\omega_0(-2T)} + U_k\, e^{-jk\omega_0 2T}$

$= U_k \left(1 + e^{jk\omega_0 2T} + e^{-jk\omega_0 2T}\right)$

$= U_k \left(1 + 2\cos(k\omega_0 2T)\right)$

$= \frac{1}{8}\,\text{sinc}\left(\frac{k}{8}\right)\left(1 + 2\cos\left(k\frac{\pi}{\cancel{4T}_2}\cdot\cancel{2T}\right)\right)$

$\boxed{S_k = \frac{1}{8}\,\text{sinc}\left(\frac{k}{8}\right)\left(1 + 2\cos\left(k\frac{\pi}{2}\right)\right)}$ reale e pari ✓

$S_0 = \frac{1}{8}\cdot 1 \cdot (1+2) = \frac{3}{8}$ ✓

Es 3

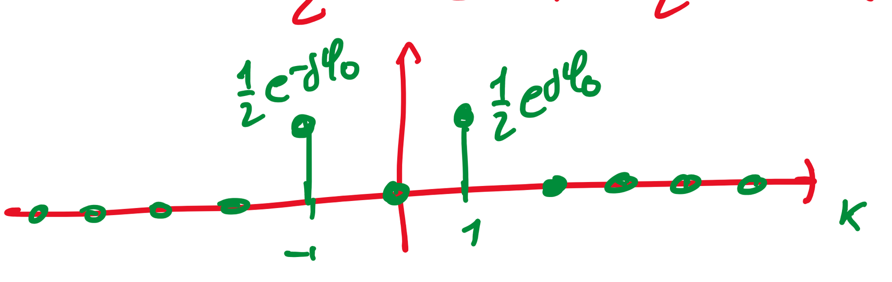
$s(t) = \cos(\omega_0 t + \varphi_0) = \frac{1}{2}e^{j\varphi_0}\underbrace{e^{j\omega_0 t}}_{m=1} + \frac{1}{2}e^{-j\varphi_0}\underbrace{e^{-j\omega_0 t}}_{m=-1}$

$S_k = ?$

$\frac{1}{2}e^{j\varphi_0} \xrightarrow{\mathcal{F}} \frac{1}{2}e^{j\varphi_0}\delta(k) \qquad \frac{1}{2}e^{-j\varphi_0} \rightarrow \frac{1}{2}e^{-j\varphi_0}\delta(k)$

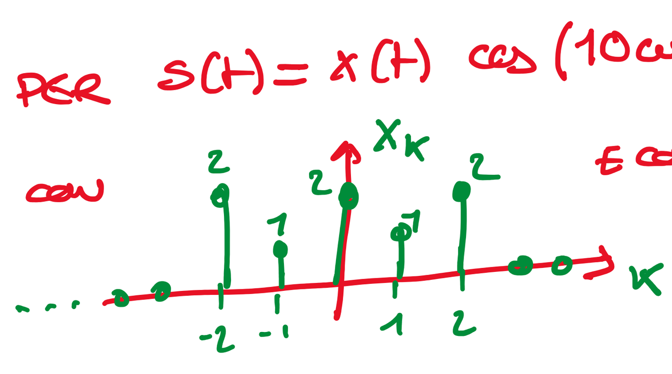
$x(t)\, e^{jm\omega_0 t} \longrightarrow X_{k-m}$

$S_k = \frac{1}{2}e^{j\varphi_0}\delta(k-1) + \frac{1}{2}e^{-j\varphi_0}\delta(k+1)$



Es 4 CALCOLARE S_k PER $s(t) = x(t)\cos(10\omega_0 t)$

CON



E CON $x(t)$ PERIODICO T_p

E $\omega_0 = 2\pi/T_p$

E TROVARE $x(t)$

$x(t) = \sum_{k=-\infty}^{+\infty} X_k\, e^{jk\omega_0 t} = 2 + e^{j\omega_0 t} + e^{-j\omega_0 t} + 2e^{j2\omega_0 t} + 2e^{-j2\omega_0 t}$

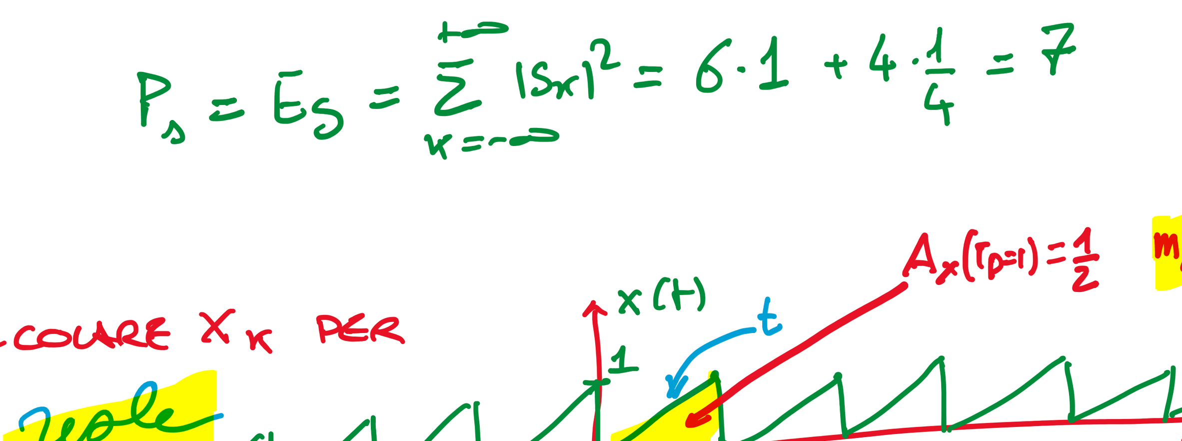
$= 2 + 2\cos(\omega_0 t) + 4\cos(2\omega_0 t)$

$s(t) = x(t)\cdot\left(\frac{1}{2}e^{j10\omega_0 t} + \frac{1}{2}e^{-j10\omega_0 t}\right)$

$= \frac{1}{2}x(t)\,\underbrace{e^{j10\omega_0 t}}_{m=10} + \frac{1}{2}x(t)\,\underbrace{e^{-j10\omega_0 t}}_{m=-10}$

$\downarrow \mathcal{F}$

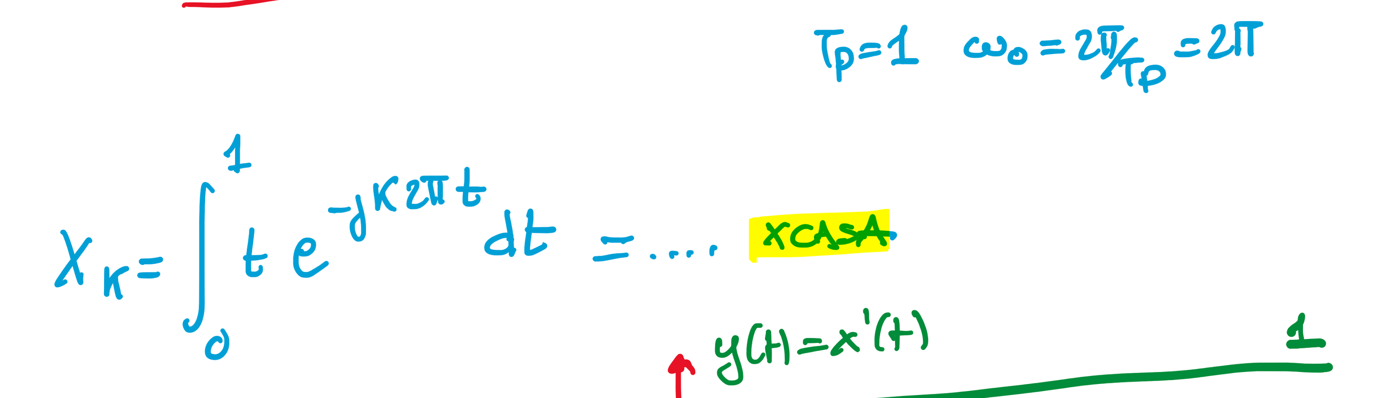
$S_k = \frac{1}{2}X_{k-10} + \frac{1}{2}X_{k+10}$



$P_s = E_S = \sum_{k=-\infty}^{+\infty} |S_k|^2 = 6\cdot 1 + 4\cdot\frac{1}{4} = 7$

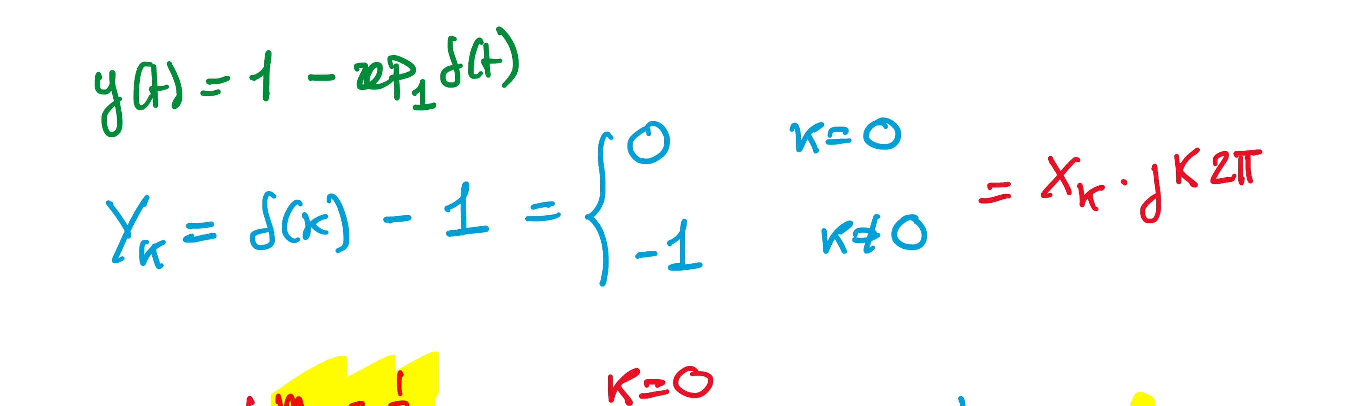
Es 5 CALCOLARE X_k PER

reale



$T_p = 1 \quad \omega_0 = 2\pi/T_p = 2\pi$

$X_k = \int_0^1 t\, e^{-jk2\pi t}\,dt = \ldots$ XCASA



$y(t) = 1 - \text{rep}_1\,\delta(t)$

$Y_k = \delta(k) - 1 = \begin{cases} 0 & k = 0 \\ -1 & k \neq 0 \end{cases} = X_k \cdot jk2\pi$

$X_k = \begin{cases} m_x = \frac{1}{2} & k = 0 \\ \frac{-1}{j2\pi k} = \frac{j}{2\pi k} & k \neq 0 \end{cases}$ Hermitiana